Chapter 8

Relativistic and Quantum Causality

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Some aspects of causality in physics are discussed. In particular, we consider the light cone in the context of special relativity, the time evolution of the wave function in quantum physics, the additional problems resulting from quantum gravity and the relevance of quantum Darwinism.

8.1 Introduction

Any scientific description of nature is inherently based on causality, since the scientific method relates observations with predictions to establish the validity of a theory. To be able to make predictions, an initial set of observables enters as input parameters into the theory; the theory itself is then used to determine uniquely a future outcome, which is only possible if the method is based on a causal chain of events. The outcome is then compared again with observations to determine the validity of the theory. In this view, causality is the property which stipulates that cause preceeds effect, i.e., that past events influence future events in a unique way. Without some form of causality a theory would lose its predictive power and, therefore, could not be considered a falsifiable theory. At the end of the 19\textsuperscript{th} century it was becoming increasingly apparent that some aspects of classical causality were not described correctly. This lead first to the development of statistical mechanics and then of the relativistic and quantum perspectives.

In addition to the notion "cause preceeds effect", the concept of causality inherently includes the fundamental laws of nature which dictate how a system evolves. It is these laws which describe the path from the initial configuration to the final outcome. Universes with different physics laws could obviously differ radically since even the same initial conditions could lead to very different final states. We do not know why the laws of physics and fundamental constants in our universe have the appearance we observe, maybe a more fundamental theory will be able to shed light on this question. On a hypothetical note, it may be conceivable that the laws of nature evolved from
an (unknown) initial state at the beginning of the universe in a causal way themselves, developing into the form they appear in at the present time.

Before the advent of quantum mechanics the belief was held that the knowledge of the positions and momenta of all the particles in the universe would permit to uniquely calculate the future (and, going causally back in time, also the past) properties of everything in the universe \[8.1\]. There are several problems with this classical picture that we would like to discuss in the following.

The first complication arises from the fact that this task would require an enormous computing machine which would itself be a part of the universe and causally interwoven with it. Even if it were possible to isolate the computing device from the rest of the universe so as to avoid causal back reactions, the computer would have to simultaneously deal with all the particles in the universe. The particles and interactions would supposedly be encoded in bits of memory in the computer, each particle, say, by at least one (quantum state of an) elementary particle out of which the computer is composed. This would render the computer comparable to the size of the universe in terms of its number of particles \[8.2\]. This goes along with the idea that the universe might itself be interpreted as a computer \[8.3\], so it is not surprising that another computer, which simulates the universe, would be of similar size.

Secondly, to compute the causal behavior of the universe we would need to know in detail all the fundamental interactions between the particles and cast them into a theory. There are indications that this may be impossible, in the spirit of Gödel’s incompleteness theorem as discussed in a lecture by S. Hawking \[8.4\]: a physical theory cannot be expected to be consistent and complete at the same time. This prevents us from calculating the future of the universe with absolute certainty.

What we may learn from the first two observations is that we can only hope to achieve the following:

(a) Either we can study the causal properties of very small subsystems of the universe to a precise level, i.e., if we pretend to extract the knowledge about the exact behavior of all the ingredients of the subsystem to a required high precision.

(b) Or we can study less precise properties of bigger systems (or the universe itself), as in statistical mechanics, which deals with average values of observables and not with the detailed behavior of each particle.

The smallness criterion relates to the Poincaré time, which is the minimum time a multiparticle system would need to return to its initial state. For macroscopic systems, which contain typically at least Avogadro’s number of particles, the Poincaré time is many orders of magnitude longer than the present age of the universe. That is, for all practical purposes a macroscopic system never returns to its initial state.

Although the evolution of the observables considered in statistical mechanics can be described by causal relations, it is very often not possible to infer from the effect what the cause has been. Even if the underlying microscopic laws are symmetric in the time variable (independent of its
sign), the second law of thermodynamics of macroscopic physics introduces an "arrow of time" defined by the time direction in which the entropy increases. As entropy increase corresponds to a loss of information about the system, causality is only realized in time-forward direction, we cannot necessarily calculate causally back in time.

Another point to mention is that there exist systems which exhibit chaotic behavior, see for instance ref. [8.5]. An infinitesimally small deviation of an initial parameter (for instance, the initial position of a particle) can lead to a completely different outcome, although all intermediate steps are well defined and causally connected. The initial coordinate cannot be determined to infinite precision due to experimental error; neither can it be represented as a real number to infinite precision in a computer, because the storage device would need an infinite number of elementary particles to represent the infinite number of digits. In consequence, the outcome of an experiment cannot be causally predicted in this kind of systems, only some general aspects of chaotic trajectories can be studied. Chaos theory and the related field of fractal structures will not be addressed in this contribution.

The next sections discuss various aspects of causality in relativity and quantum physics. The following presentation should be regarded as an outline and is in no way meant to be complete.

Section 8.2 deals with the relativistic perspective which has an important impact on causality. The need for a relativistic theory was recognized when Maxwell's equations of electrodynamics were formulated: they are invariant under Lorentz transformations and not under Galilei transformations as the equations encountered in classical Newtonian physics. The reformulated, Lorentz invariant classical theory is Einstein's theory of special relativity, and the inclusion of gravity resulted in the theory of general relativity.

In sections 8.3 and 8.4 we consider quantum physics. At the turn of the 19th to the 20th century it was found that dynamics at the molecular and atomic level is indeed different from classical Newtonian physics. This led to the formulation of quantum mechanics. In quantum physics, the classical concept of a well-defined trajectory fails at the quantum level, because the exact momentum and position of an elementary particle cannot be known to arbitrary precision by fundamental reasons.

Section 8.5 discusses quantum Darwinism. In Chaitin's perspective of Gödel's incompleteness the amount of Shannon information plays a critical role in explaining the reason for the incompleteness. In quantum Darwinism the axioms of quantum mechanics are re-defined so as to have the transported information be a key component of the quantum process. Quantum Darwinism only makes predictions different from standard quantum mechanics in the cases where the amount of information transported during a measurement is small but non-zero, it is thus more a fine-tuning of standard quantum mechanics than a new form of quantum theory. One of the consequences of quantum Darwinism is the balance between observing the whole system versus observing parts of the system, in agreement with Hawking's inferences from Gödel's incompleteness, especially when considering Chaitin's information-based approach to Gödel's incompleteness. It is possible that quantum Darwinism will be helpful in obtaining a successful formulation of quantum gravity.
The last section contains a brief summary and concluding remarks.

8.2 Causality and the light cone

When studying the causal evolution of a system we often deal with velocities and accelerations of particles or with the velocity of propagation of radiation. A fundamental question is: with respect to what do we define velocities and accelerations? In their famous 1887 experiment, Michelson and Morley [8.6] tried to determine the velocity of the Earth with respect to the "aether", the hypothesized medium through which light propagates. The only consistent interpretation of their result is that the speed of light \(c\) is constant in any referential frame, meaning that an "aether" does not exist. These observations, together with the Lorentz invariance of the Maxwell equations, led to the development of the theory of relativity. Velocities become a relative concept, all frames of reference moving with constant relative velocities are inertial frames where no forces due to acceleration are felt. But if there is no "aether", how does one define acceleration? Consider for instance a bucket filled with water. According to Newton's laws, the water in a rotating bucket (rotation is a form of accelerated motion) is pushed towards the bucket walls, thus curving the water's surface. Imagine, however, an empty universe, how do we know if the bucket is rotating or not? In the more realistic case of a universe filled with galaxies, if all the galaxies rotate around the bucket but the bucket is at rest, would the water surface curve? Classical Newtonian physics would say no; in the converse situation of the galaxies at rest and the bucket rotating, the surface would be expected to curve. Again we can ask, with reference to what can we determine if it is the galaxies which rotate or if it is the bucket, given that there is no "aether"? Mach's principle states that the water surface in the bucket curves if the bucket rotates with respect to the background of the galaxies in the universe. This is the point of view of general relativity, where physics evolves locally on the globally curved spacetime which is generated by the gravitational fields of all the matter distribution in the observable universe. To sum it up, although there exists no "aether", there does exist a preferred referential frame; it is the frame in which the galaxies are, on average, at rest. The absolute velocity of the solar system with respect to this frame is directly observable by measuring the Doppler shift of the cosmic microwave background radiation [8.7]; it is about 370 km/s.

We know from the theories of special and general relativity that the highest speed a signal or a particle can travel is, in any reference frame, the speed of light \(c\). This is apparent in the Lorentz transformation equations which relate the space and time coordinates \(\mathbf{x}, t\) defined in an inertial system \(S\) with the coordinates \(\mathbf{x}_0, t_0\) defined in another inertial system \(S_0\). (As discussed before, inertial systems are reference frames in which no forces due to acceleration or gravitational fields exist. In the absence of gravitational fields, i.e. in "flat spacetime", inertial frames all have constant velocities with respect to each other). The two systems move away from each other with a uniform velocity \(v\). Without loss of generality we can choose the orientation of the reference frames in such a way that the relative motion is along the \(x\)-axis, as depicted in Fig. 8.1. Then the Lorentz transformations are

\[
x_0 = \gamma(x - vt), \quad y_0 = y, \quad z_0 = z, \quad t_0 = \gamma(t - \frac{v}{c^2}x)
\] (8.1)
Figure 8.1: Inertial frames.

where

\[
\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]  

(8.2)

One sees that the factor \( \gamma \) goes to infinity when \( v \) approaches \( c \). It can be shown that the energy necessary to accelerate a particle to the speed of light goes to infinity as well, since \( E = \gamma m_0 c^2 \) where \( m_0 \) is the rest mass of the particle. The limiting speed leads to the concept of the light cone. An observer located at the origin of the coordinates in Fig. 8.2 can only have causal influence on events which appear within the "future light cone" and only be influenced by past events which lie in the "past light cone". Note that for simplicity Fig. 8.2 only represents 2-dimensional space (plus time); in reality space is of course 3-dimensional, so the light cone is mathematically a hypercone in 4-dimensional spacetime which is difficult to visualize.

Figure 8.2: Past and future light cones.

In 2011, the Opera collaboration working at CERN and Gran Sasso claimed to have observed superluminal neutrinos [8.8]. Although this turned out later to be an error of a loose cable in the experiment, such an observation would be disastrous for physics in general, as it would violate
causality. To see this, consider the Lorentz transformations above. We now perform an experiment in $S$, where a particle or signal is sent a distance $\Delta x$ along the $x$-axis, from initial point $A$ (the "cause") to point $B$ (the "effect"), but with a superluminal speed $w > c$, a so-called tachyon. In $S$, the corresponding time interval is

$$\Delta t = t_B - t_A = \frac{x_B - x_A}{w} = \frac{\Delta x}{w}. \quad (8.3)$$

How does the experiment appear in $S_0$? From the Lorentz transformation $t_0 = \gamma(t - vx/c^2)$ we get

$$\Delta t_0 = \gamma(\Delta t - \frac{v}{c^2}\Delta x) = \gamma\Delta t(1 - \frac{v}{w}/c^2) . \quad (8.4)$$

We can write $w = ac$ with $a > 1$ since $w > c$, and $v = c/b$ with $b > 0$ (for a "well-behaved" pair of reference systems with relative velocity $v < c$, one has $b > 1$ also). Then if we choose an inertial system $S_0$ such that $b < a$, the time interval observed in $S_0$,

$$\Delta t_0 = \gamma\Delta t(1 - \frac{a}{b}) < 0 , \quad (8.5)$$

becomes negative. As seen in $S_0$, "effect" precedes "cause", a disaster for causality.

The light cone is also interesting in the study of black holes. A black hole has a gravitational field so strong that even light cannot escape: from a certain proximity from the center of the black hole on the light cones in curved spacetime, as described by general relativity, are tilted towards the center of the black hole in such a way that no light ray can leave this so-called event horizon, see Fig.8.3. In the case of a non-rotating uncharged black hole the event horizon is the surface at the Schwarzschild radius $R_S$. For a black hole with mass $m$ the Schwarzschild radius is

$$R_S = \frac{2Gm}{c^2} . \quad (8.6)$$

Figure 8.3: Schematic representation of the tilting of light cones of a light source falling towards a black hole. For distances closer than the Schwarzschild radius $R_S$, no light can escape towards an observer at infinity.
where $G$ is the gravitational constant. Since nothing can escape but matter and radiation are allowed to enter, the mass of a classical black hole can only increase (or stay constant). It has also been shown that the only properties of the black hole that an outside observer can measure are its total mass, electric charge and angular momentum [8.9]. Since the particles and radiation which enter the black hole have more known properties than that, it was realized that information is lost in the process and that the black hole’s surface area can only increase (or stay constant) [8.10] and is proportional to entropy [8.11], as entropy increase is a measure of loss of information. With respect to causality this means that processes near a black hole can end up in a causal dead end, that is, it is possible to calculate causal connections towards the future but not necessarily back in time, as in the situation encountered in statistical mechanics.

8.3 Causality and the wave function

Let us now turn to quantum physics. The impossibility to determine the precise position and momentum of a quantum particle can be expressed as Heisenberg’s uncertainty relation

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

(8.7)

where the quantities $\Delta x$ and $\Delta p$ refer to the uncertainty in position and momentum, respectively, $\hbar$ being the Planck constant divided by $2\pi$. The more precisely one determines the position of a particle, the more uncertain becomes the knowledge of its momentum and vice-versa. One consequence is that a free particle occupies a “phase space cell” (the product of position times momentum) of the order of the Planck constant cubed (in three dimensions).

Since position and momentum cannot be determined to arbitrary precision at the same time, the classical concept of a trajectory of a particle breaks down. The experimental evidence for this behavior is schematically depicted in Fig.8.4. The particle source on the left emits e.g. electrons or photons which then pass through a double slit and impact the screen on the right where they are detected. If the particle flux is low, one can observe that the particles hit the screen at points which are distributed in a seemingly random way. Accumulating enough of these events, however, produces a diffraction pattern with clear minima and maxima, i.e. regions in which less or more particles accumulate. That is, although it is impossible to describe the point at which a single particle will hit the screen, quantum mechanics allows us to calculate the probability with which a certain point on the screen is hit. The emerging diffraction pattern is equivalent to diffraction in wave optics. Quantum mechanics, therefore, does not predict the exact trajectory of a particle, but it predicts probabilities, which are the absolute squares of probability amplitudes or wave functions. The wave functions are also used to calculate observables, which are represented as expectation values of mathematical operators, i.e. integrals containing the wave functions. Wave functions are the objects which appear in quantum mechanical equations of motion and are subject to causal, deterministic evolution.

Moreover, it is impossible to determine through which one of the two slits a particle passes. In fact, to produce the diffraction pattern, the particle has to pass through both slits. Any experimental setup which allows to detect through which slit the particle passes alters the diffraction pattern, much like closing one of the slits in a wave optics experiment. This reflects again the breakdown
of the concept of a trajectory. A particle released at point A and freely propagating to B where it is detected, passes through all possible intermediate points, which can be represented by multiple slit gratings at multiple positions (Fig. 8.5). In reality, the number of slits goes to infinity, as well as the number of diffraction gratings, whose spacing goes to zero so as to cover the whole space. This is the concept behind Feynman’s path integral technique \[\text{(8.12)}\], where every possible path has to be summed over since it is not measurable. Each path contributes with the same weight but with a phase factor which contains the ratio of the action \( S \) divided by Planck’s constant \( \hbar \) in the exponential; it is this phase factor which selects the classical path as the most probable in classical systems: in classical physics the action \( S \) is much larger than \( \hbar \) (which has the units of an action), so formally the classical limit of quantum mechanical expressions is achieved by letting \( \hbar \to 0 \). In this limit, quantum uncertainty disappears and the classical trajectory is the only one which survives, as it can be shown that all the other paths cancel one another.
which quantum mechanics considers are the wave functions and derived expressions. Their time
evolution, as described by the Schrödinger equation in the nonrelativistic case or by the Klein-
Gordon equation or Dirac equation in the relativistic case, is deterministic and uniquely given by
the initial conditions and is in this way causal.

Is it possible to go beyond the probabilistic description which quantum mechanics provides? In
a famous paper by Einstein, Podolsky and Rosen \[8.13\] it is argued that quantum mechanics is
not a complete theory. This argument suggests the existence of hidden parameters which are
not being contemplated in quantum theory. The gedankenexperiment contrived is called the EPR
paradox. A pair of particles is prepared in an entangled state. Entanglement is the property
which determines the outcome of a measurement of the second particle when the measurement
result of the first one is known, but before a measurement is performed all possible results allowed
by quantum mechanics can be expected. The pair of particles then leave the source in opposite
directions. The paradox is then that the outcome of the measurement on the first particle in point A
has an immediate effect of the outcome of a distant measurement in point B of the second particle
in a spooky action at a distance (Einstein). Either the second particle knows from the beginning
more than quantum mechanics describes (hidden variables) or a signal travels with faster than
the speed of light (actually at infinite speed) from A to B to inform the second particle of the
outcome of the measurement of the first. Signals traveling faster than the speed of light lead to
violation of causality as we have seen, so this interpretation cannot hold. Indeed it can be shown
that the observer in A, when performing his measurement, does not and cannot send a signal to
B, no superluminar information exchange exists. The paradoxical nature of the experiment stems
from the classical picture of locality, the property that an event can only have immediate effects
on its immediate surrounding. Entangled quantum systems are not local in this sense, but in no
experiment superluminar information exchange could be observed. Moreover, Bell's paper \[8.14\]
shows that the outcome of certain experiments is different in the presence of hidden variables as
compared with standard quantum mechanics. All experiments performed so far show the validity
of quantum mechanics and therefore exclude hidden variable theories, see e.g. \[8.15\].

An additional level of complexity arises in quantum field theory, the theoretical quantum frame-
work to describe subatomic particle physics. Standard quantum mechanics uses the (actually
classical) notion of the forces acting on particles being represented by potentials. Quantum field
theory describes (more correctly) the forces as being due to interactions. The interactions are
mediated by fields which exhibit quantum behavior themselves. That is, wheras standard quantum
mechanics allows a (quantum) particle to move in a fixed background of potentials, this classical-
type backround cannot really exist since it has quantum properties on its own and produces a
new source of uncertainty.

It seems thus that all that microscopic physics can do is to describe the evolution of probability
amplitudes in a deterministic way, but it cannot predict a unique outcome of an experiment, by
fundamental principles. Causality is observed, but only at the level of probability amplitudes.
8.4 Quantum gravity

In many situations it is necessary to simultaneously apply the concepts of general relativity and quantum physics. A large amount of work has been invested in trying to unify these two aspects of physics in a common framework, the theory of quantum gravity. Loop quantum gravity, string theory and branes are some of the candidate theories. It is fair to say that the final goal to achieve a unified theory has not yet been reached. Many aspects can be discussed however in an approximate way, for instance by embedding the equations of quantum mechanics in curved spacetime (the geometry of space and time in the presence of gravitational fields or accelerations).

To see the necessity of the unification of quantum physics and general relativity, let us study the distance scales to ever increasing resolution, i.e. decreasing the length scale. One quantity to consider is the Schwarzschild radius $R_S = \frac{2Gm}{c^2}$; it encodes a fundamental gravitational aspect of an object. The quantum physical aspect of the object is expressed in the Compton wavelength of a particle with dynamical mass $m$

$$\lambda_C = \frac{h}{mc},$$

which in turn is the de Broglie wavelength

$$\lambda = \frac{h}{p}$$

of an ultrarelativistic particle with momentum $p = E/c = mc$, since the relativistic energy of a particle with rest mass $m_0$ in motion is $E = \sqrt{p^2c^2 + m_0^2c^4}$ and ultrarelativistic means that $pc \gg m_0c^2$, so the energy becomes $E = pc$. The length scale at which the Schwarzschild radius $R_S$ is equal to the Compton wavelength $\lambda_C$ is called the Planck length

$$l_P = R_S = \lambda_C = \sqrt{\frac{2Gh}{c^3}} \simeq 5.7 \times 10^{-35} \text{m},$$

and the corresponding time scale is the Planck time

$$t_P = l_P/c = \sqrt{\frac{2Gh}{c^5}} \simeq 1.9 \times 10^{-43} \text{s}.$$ 

This leads to the observation that at this small scale quantum physics and general relativity become fundamentally interwoven.

There are many consequences resulting from this consideration. For one, the light cone becomes fuzzy at the Planck scale, since general relativity has to take into consideration the inherent Heisenberg uncertainty of quantum mechanics. The immediate question arises if causality is violated when studying processes at the Planck scale or below. Since our current experiments do not test physics down to such small scales it is possible that causality violations do occur at this scale. These eventual small-scale violations should however behave in such a way as to have no consequences on larger scales, where causality is observed to be valid.

Another consequence of quantum gravity is that the mere concept of the number of particles is frame dependent if non-inertial reference frames are involved; this is the Unruh effect [810]. Consider,
as an example, a monochromatic massless scalar field in an inertial frame (units $\hbar = c = 1$),

$$\phi(x, t) = \frac{1}{\sqrt{2E}} e^{i(px-Et)} = \frac{1}{\sqrt{2E}} e^{iE(x-t)}$$  \hspace{1cm} (8.12)

because $E = \sqrt{p^2 + m^2} = p$ in the massless case, the square root in the denominator being the correct relativistically invariant normalization factor. When transforming to a reference frame which is uniformly accelerating with respect to the inertial system, the plane wave becomes

$$\phi(\tau) = \frac{1}{\sqrt{2E}} e^{iE(cosh(g\tau) - sinh(g\tau))},$$  \hspace{1cm} (8.13)

where $g$ is the acceleration and $\tau$ is the time coordinate in the accelerated frame. To obtain the spectral content of the field as observed in the accelerated frame, a Fourier transform $\phi(\tau) \rightarrow f(\omega)$ is performed and the spectral content is calculated as

$$|\sqrt{2\omega}f(\omega)|^2 = \frac{2\pi}{Eg} \frac{1}{e^{2\pi\omega/g} - 1},$$  \hspace{1cm} (8.14)

which is the Planck spectrum with temperature $T = \frac{g}{2\pi}$. Therefore, what is a monochromatic wave in one system (single energy $E$), is a thermal spectrum of frequencies $\omega$ as observed in the other. Instead of observing a single particle, as in the inertial frame, a whole spectrum of particles is observed in the accelerated frame. If a causal scheme is to describe also the number of particles which exist, are produced or are annihilated one has to define exactly in which reference frame the description proceeds. This may be difficult because, in general relativity, accelerated systems are indistinguishable from systems subjected to gravitational fields (this is the equivalence principle). Since gravitational fields pervade the universe, an experiment which is not strictly local will "feel" different gravitational fields as particles propagate. This corresponds to changing reference frames. When gravitational fields are not weak and therefore spacetime not flat, the observations should suffer from the causal ambiguities of particle number indeterminacy.

It is not surprising that the Unruh effect together with the equivalence principle of general relativity give rise to the claim that black holes emit Hawking radiation [8.17]. Whereas classical general relativity does not allow anything to escape from the black hole and the black hole can only increase in mass by absorbing surrounding matter, quantum effects predict that an observer far away (where gravitation is weaker) detects particles produced near the black hole horizon (where gravitation is stronger). The energy required to produce the particles is extracted from the black hole, reducing its mass. Does entropy then decrease in such a process? It was shown that the decrease of the entropy of the black hole is more than compensated by the entropy of the produced Hawking radiation, so total entropy never decreases [8.18]. Again, entropy increase interpreted as a measure of information loss makes causality work only in time-forward direction.

8.5 Darwinism in quantum physics

In previous chapters we have addressed two major forms of causality that are being used to describe the observable world: Newtonian, and Darwinian causality. Possibly the first to discuss the relation and antagonism between Newtonian and Darwinian causality was Kant, even before
Darwinism was developed. In the third Critique \[8\] § 80-81 Kant envisages the understanding of nature as a unified whole. Paragraphs § 80-81 propose firstly that, through comparative anatomy and the analogical study of living forms, one can firmly assume an "effective universal kinship" tying them together, and suppose their generation from a "common original model" (Urbild) or "original mother" (Urmutter) - presaging Darwin’s "tree of life". Secondly, the underlying principle of continuity among all living forms is applied to the generation of biological novelty, in such a consistent way that Kant considers schematically different types of generation (generatio aequivoca versus univoca, and homonyma versus heteronyma) and examines some defining features of the intra-, inter- and trans-specific generative solutions, worked later out in detail by Lamarck [8\[20\]] and Darwin [8\[21\]]. Thirdly, Kant extends the principle of continuity and communality from the various kingdoms of organic beings to the domain of lifeless matter, in an attempt to unify all expressions of finality and find out the location for the "first production of something containing ends in itself and being intelligible solely through those ends".

Quantum Darwinism was proposed by Zurek as a means for better describing the transition between quantum and classical mechanics [8\[22\]]. The advantage of quantum Darwinism over other approaches to quantum mechanics is that it is not based on a postulate of the existence of a classical world, unlike the Copenhagen interpretation; this fact allows for a smooth transition between quantum and classical rather than a sharp qualitative transition. The Copenhagen interpretation is for now the most widely used interpretation despite its several flaws that quantum Darwinism seeks to address. In quantum Darwinism all aspects of the universe are governed by the laws of quantum mechanics. As the name indicates, quantum Darwinism borrows considerably from the concept of natural selection.

Zurek's postulates of quantum Darwinism in simplified form are the following [8\[23\]]:

\(o.\) The universe consists of systems individually accessible to observations (defined as the effect a system has on its exterior).

\(i.\) The state of a system is represented by a vector in a multidimensional vector space. To this vector space corresponds a dual vector space such that to each element of the vector space there corresponds an element of the dual vector space; and to any pair of vectors (the first a vector and the second a dual vector) there corresponds a unique scalar. This value is called the inner product of the two vectors. In addition, all convergent series of vectors in the vector-space converge to a vector that is also a part of that vector space.

\(ii.\) The time evolution of a vector is such that the inner product of a vector is preserved.

\(iii.\) If the outside of a system remains unchanged, then the observation of the system remains unchanged in time.

Postulate \(o\) establishes the universe as being constituted by systems observed by other systems, this relational perspective of the universe extends the relativity of space–time to the relativity of system states. Postulate \(i\) considers that the appropriate form for systems to represent other systems it observes is through vector spaces. These vectors are considered to be arbitrarily re-scalable, therefore they can be rescaled to unit length, meaning that each vector corresponds to a point on a unit sphere of a space with an arbitrary number of dimensions. This space is called a Hilbert space. Postulate \(ii\) establishes a property which is preserved during evolution,
implying that some property in the past has the capacity to determine some property in the future; it is thus a form of including a certain amount of Newtonian determinism. Postulate \(iii\) establishes that stability of the outside of a system implies the stability of an observation about the system, implying that environmental stability allows for observational stability, a concept very akin to environmental stability influencing speciation stability in Darwinian evolution. In Quantum Darwinism, the universe is relational, mappable, Newtonian and Darwinian; however, space and time are kept separate, an inconvenience for successfully dealing with quantum gravity that we will assess at the end of this chapter.

In previous paragraphs we represented quantum fields as being \(\phi(x, t)\), for example; note that in such a function the basis for the representation of the field has already been selected, in this case space \(x\) and time \(t\). In statement \(i\), however, the maps chosen are not made specific and so there must be a way for representing a function without specifying its representation basis, call it \(\vec{\phi}\). Each value of the function is a complex number, so if there is a dual vector made of the possible space points at a time \(t\), call it \(\vec{\phi}(x, t)\) the inner product between the space-points dual vector and \(\vec{\phi}\) becomes

\[
\phi(x, t) = (\vec{x}(x, t) \cdot \vec{\phi}).
\]  

If one considers the existence of multiple quantum fields, e.g. \(\vec{\phi}\) and \(\vec{\psi}\), the vector representing that sum would be \(\vec{\phi} + \vec{\psi}\). A historical and essential assessment of the concept of randomness is that provided by Laplace's "card swapping" \([8.1]\). We will thus consider, following Zurek's example \([8.23]\), that it is useful to represent these vectors not by greek symbols but by card suits. Let us consider system \(S\) surrounded by a set of systems that we call "the environment", \(E\) that is capable of making one-to-one measurements of \(S\). According to postulates \(o\) and \(i\) the following statements can be made \([8.23]\):

A) Let the state of the system \(S\) (with \(\alpha\) and \(\beta\) scalars different from zero) be

\[
\vec{S} = \alpha \vec{\spadesuit} + \beta \vec{\heartsuit}.
\]  

B) Let the state of the environment \(E\) before interacting with \(S\) be

\[
\vec{E}_0 = \vec{\diamondsuit}_0 + \vec{\clubsuit}_0.
\]  

C) The state of the environment \(E\) after beginning to interact with \(S\) is then

\[
\vec{E} = \vec{\diamondsuit} + \vec{\clubsuit}.
\]

Due to postulates \(ii\) and \(iii\), the evolution of the state of the environment must be such that

\[
\alpha \beta^* \vec{\spadesuit} \cdot \vec{\heartsuit} \left( |\vec{\diamondsuit}_0 + \vec{\clubsuit}_0|^2 - \left( \vec{\diamondsuit} \cdot \vec{\clubsuit} \right) \right) = 0,
\]  

which means that either \(\vec{\diamondsuit} \cdot \vec{\clubsuit}\) remains a constant, implying that the system has no influence on the environment; or \(\vec{\spadesuit} \cdot \vec{\heartsuit} = 0\), implying that the two vectors representing the system are orthogonal.

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Thus, it follows from the four postulates of quantum Darwinism [8.23] that the only way the state vector of a system interacting with an environment can evolve is that either the inner product of any vector state of the system with any vector state of the system different from itself is zero (meaning the two states are orthogonal), or that the vector state of the system has absolutely no influence on the environment. This guarantees that the vector states of the system capable of leaving an imprint in the environment, no matter how small the imprint, will be orthogonal vector states.

The interaction with the environment makes the states of the system that can transmit information into the environment be orthogonal, but once the transmission of information about the system to the environment makes the vector states of the system be orthogonal, the vector states of the environment associated with the corresponding states of the system can themselves be orthogonal. In the case where the environment vector states are themselves orthogonal, the environment is said to have a perfect record of the system.

The orthogonality of the vector states capable of being measured in a certain environment strikes a balance between the interchangeability among states expressed in statements $i$ and $ii$ with the need for a certain state to be able to preserve its existence after measurement expressed in $iii$. The preservation of the information about the measurement in the environment in large enough quantities to be read by multiple observers, meaning the “Darwinian” survival of that information after the measurement has occurred, is what creates the illusion of state “objectivity”, even if we are only able to interact directly with the environment and never with the system itself.

In quantum Darwinism, the “reality” of the vector states of the system is given by their capacity to transmit information into the vector states of the environment. That, is the “reality” is an outcome of the capacity of the system to print copies of its orthogonal vector states into the environment, combined with the capacity of the environment to define which set of orthogonal vector states the system will be expressed on. The successful transmission of information is the imprinting of the information about the system into the environment states. The vector states before the “collapse” caused by the interaction with the environment have a certain ontic aspect to them as they are all that the system is (meaning all that can be said about the system) before the measurement; and the state of the system after the measurement has a certain epistemic aspect to it as the system does not completely adopt an objective existence, but it only does so in as much as it is capable of leaving imprints about itself in the environment. These vector states can therefore be called epiontic states.

A fully known\footnote{A “fully-known system” in quantum Darwinism is equivalent to the concept of “system in a pure state” in the Copenhagen interpretation of quantum mechanics.} entangled (meaning linked) combination of system and environment can be described by a vector state where each component of the vector is the product of a system output state $S_k$ with its corresponding environment output state $E_k$. The coefficient associated to that product of output states is called the Schmidt coefficient, $\alpha_k$, and is a complex number\footnote{For the notion of Schmidt coefficient, cf. [8.24]}. According to statement $i$, the universe consists of systems; statement $i$ makes it explicit that the state vector is the representation of the system; and statement $ii$ describes the change of the state vector of a system in a form that preserves the inner product. Combining the statements $i$-$iii$ it is clear that...
a transformation of a system must act on the vector space representing that system to affect the system; the vector space of a system larger than $S$ which includes $S$ is all that is needed and available to describe the state of the system $S$; and the state of the system $S$, given the measurements already performed, is all that is needed and available to predict measurement results.

One possible type of measurement is the following: we choose to detect all that we can about the environment, in which case we obtain the value associated with the link between the system and the environment. In Copenhagen quantum mechanics this corresponds to measuring an observable. Another type of measurement is the choice to learn all that we can about the system, and thus necessarily block the system's link to the environment. In that case all that we can measure is the probability of the outcomes, which in Copenhagen quantum mechanics corresponds to the wave function of the not-measured system.

Since, like for all complex numbers, $\alpha_k = |\alpha_k|e^{i\theta_k}$, one obtains that the action of transforming system $S$ cannot change the amplitude of the output states of either the system or the environment because they must be compatible with eq. (8.19), implying that what is observed about the system must be in direct relation to orthogonal vector states and the transformation cannot change the amplitude of the coefficients because it must be a unitary transformation as described in postulate $ii$. The same can be said of a transformation acting on the environment $E$. This implies that for an entangled system–environment state an action on the system causes a change of the phase $\theta_K$ of the Schmidt coefficient, written as $\alpha_k = |\alpha_k|e^{i\theta_k}$, and that change can be compensated by a change of the environment. This means that it is possible to describe a kind of invariance where a system is altered by a transformation, but where that alteration of the system can be nullified by an action on that system's environment, without the occurrence of any further action on the system; this type of invariance is called an environment-assisted invariance, which in short form is called \textit{envariance}.

If the entangled (meaning linked) "$S$ and $E$" state is \textit{envariant} under a certain transformation acting solely on $S$, then the state of the system $S$ must be invariant under that transformation because the state of a system which includes $S$ is all that is needed and available to describe the state of the system $S$. Therefore, in a fully known entangled system–environment state, the state of $S$ is invariant under changes of the Schmidt coefficient phase $\theta_K$ associated to each outcome state; and likewise for the state of the environment $E$. So the state of $S$ on an entangled "$S$ and $E$" state can only depend on the amplitude of the Schmidt coefficient amplitudes $|\alpha_k|$ of the outcome state $\vec{S}_k$ itself. The entanglement is thus a selective loss of the relevance of phases for the state of $S$, hence a consequence of the onset of envariance, and it is a process that occurs gradually through time by the flow of information between the system and the environment, and not the result of a drastic change.

If the outcome states of a system–environment entangled system have the same Schmidt coefficient amplitude, $|\alpha_k|$, then they are swappable. The concept of swappable is based on Laplace's "principle of indifference" [8.1]. Figure 8.6 is an illustration of that principle as interpreted by Zurek [8.23]. The concept of swappable states implies that their labelling/ordering/naming is arbitrary; arbitrariness is thus objectively defined as equality of the Schmidt coefficients. Because the output states of $S$ and $E$ are perfectly correlated one to one, the probability for each of them must be equal, since they are swappable. More precisely, after two outcome states are swapped in $S$ by a transformation not acting on $E$, their new probabilities must be the same as the prob-

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abilities of their new $E$ partners, but the $E$ states were not altered since the transformation did not alter $E$. So the probabilities of the two $S$ states must be unchanged by the swap, meaning that they must be identical. Thus, probabilities of envariantly swappable states are equal. When all countable (meaning they are either finite, or infinite but ordered) $N$ Schmidt coefficients have the same amplitude, the probability of the output states associated with each Schmidt coefficient is the same, and it is equal to $\frac{1}{N}$ due to the requirement of normalization. Associated to each Schmidt coefficient, $\alpha_K$, there is a system and an environment output state, which thus have a probability of $\frac{1}{N}$ of occurring when measured.

![Diagram](image)

**Figure 8.6:** Probabilities and symmetry: (a) Laplace (1820) appealed to subjective invariance associated with 'indifference' of the observer based on his ignorance of the real physical state to define probability through his principle of equal likelihood. When the observer's ignorance means he is indifferent to swapping (e.g., of cards), alternative events should be considered equiprobable. (b) The real physical state of the system is however altered by the swap, it is not ‘indifferent’, illustrating the subjective nature of Laplace's approach. The subjectivity of equal likelihood probabilities posed foundational problems in statistical physics and led to the introduction of imaginary ensembles. (c) Quantum Darwinism allows for an objective definition of probabilities based on a perfectly known state of a composite system and symmetries of entanglement: When two systems ($S$ and $E$) are maximally entangled (i.e, Schmidt coefficients differ only by phases, as in the Bell state above), a swap in the system $S$ can be undone by a ‘counterswap’ in the environment $E$. Reproduced from [8.23] with kind permission of the author.

The phases of the system states lose their importance due to the entanglement caused by the selection of states that are capable of printing information about them in the environment. This loss of the importance of the phases is responsible for the sum of the probabilities of measurement occurrences to make sense. The sum of state vectors loses its meaning since the phase of each of its basis states $S_k \otimes E_k$ has become objectively arbitrary. This arbitrariness implies that the only states that can affect the environment are those that are orthogonal to each other, meaning that the only states capable of making an imprint in the environment are those orthogonal between themselves.
Our proposal is that in quantum Darwinism this interaction between Newtonian determinism and Darwinian determinism is possible because of the transcendental randomness ("transcendental" meaning here non-calculable or non-computable) of quantum events as expressed in the experimental breaking of Bell's inequality. A Newtonian determinism is a process where the past defines the future, and a Darwinian determinism is a process where future states exist not only because of their past, but because of their capacity for establishing a positive interaction amongst themselves. Newtonian and Darwinian causality are hence complementary aspects of axiomatic systems, corresponding to two extreme forms of dealing with Gödel's incompleteness, the first choosing consistency and the second completeness, and which can only interact if intrinsic (at once noumenal and phenomenal) randomness exists. Kant did not believe it would be possible for true randomness to exist, but it does exist, as Leibniz expected. It is that true randomness allows for the causality of the state's evolution to be partially dissociated from the apparent causality of the observed variables. Because of the true randomness, the causality of the states is not stringent enough to generate a causality of the observed variables, with states not generating observed variables becoming effectively extinct.

8.6 Conclusions

To summarize this chapter, we first discuss the light cone which limits the region within which causal processes unfold and define the global reference frame of physical processes, as expressed in Mach's principle: the background of all the galaxies in the observable universe. The behavior of the light cones close to a black hole represents one of the physical situations in which causal connections can be followed forward in time but not necessarily backwards.

We then proceed to show that although quantum physical systems do evolve obeying causality, they do so at the level of probability amplitudes and not in the classical sense of precisely defined trajectories. There is therefore an intrinsic randomness in quantum physics and results can only be described in a probabilistic interpretation. Quantum gravity leads us to possible violations of causality at a very small distance scale, the Planck scale, and to the frame dependence of the measurement of particles as expressed in the Unruh effect and Hawking radiation. The entropy increase associated with this radiation corresponds to information loss which makes causality work only in time-forward direction in these processes.

In the last part we discuss the quantum Darwinism perspective of how the classical world emerges from the underlying quantum constituents. Although quantum systems evolve causally in the way described above, not all possible quantum states "survive" the selection process imposed by the interactions with the macroscopic environment. Since the discussion presented here is based on Schrödinger-type standard quantum mechanics it would be interesting to formulate quantum Darwinism in the more fundamental context of quantum field theory. A possible approach would be to implement the Darwinian selection mechanism as extinction of paths in the Feynman path integral. Work in this direction is in progress.

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