Classification of $a_0(980)$ and $f_0(980)$ within the family of scalar mesons

prepared for

HADRON PHYSICS at COSY

July, 7-10 (2003)

Physikzentrum, Bad Honnef (Germany)

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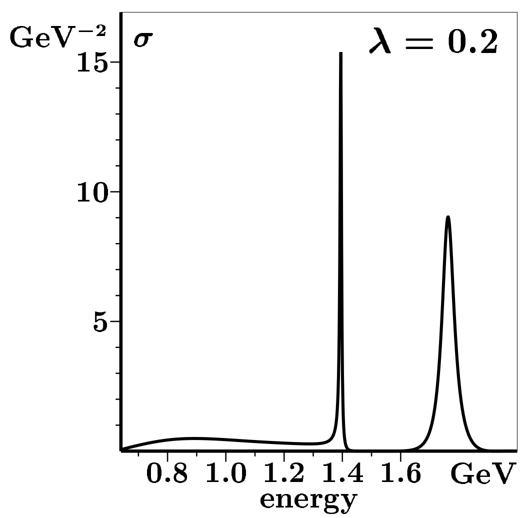
Miss QCD
and her little fellow Electroweak
Bad Honnef, July 9, 2003

Miss QCD, now in her early thirties, embarrassed by the very thought that from the not-exactly-faithful candidates, soon she should select a partner for life.

Warming up

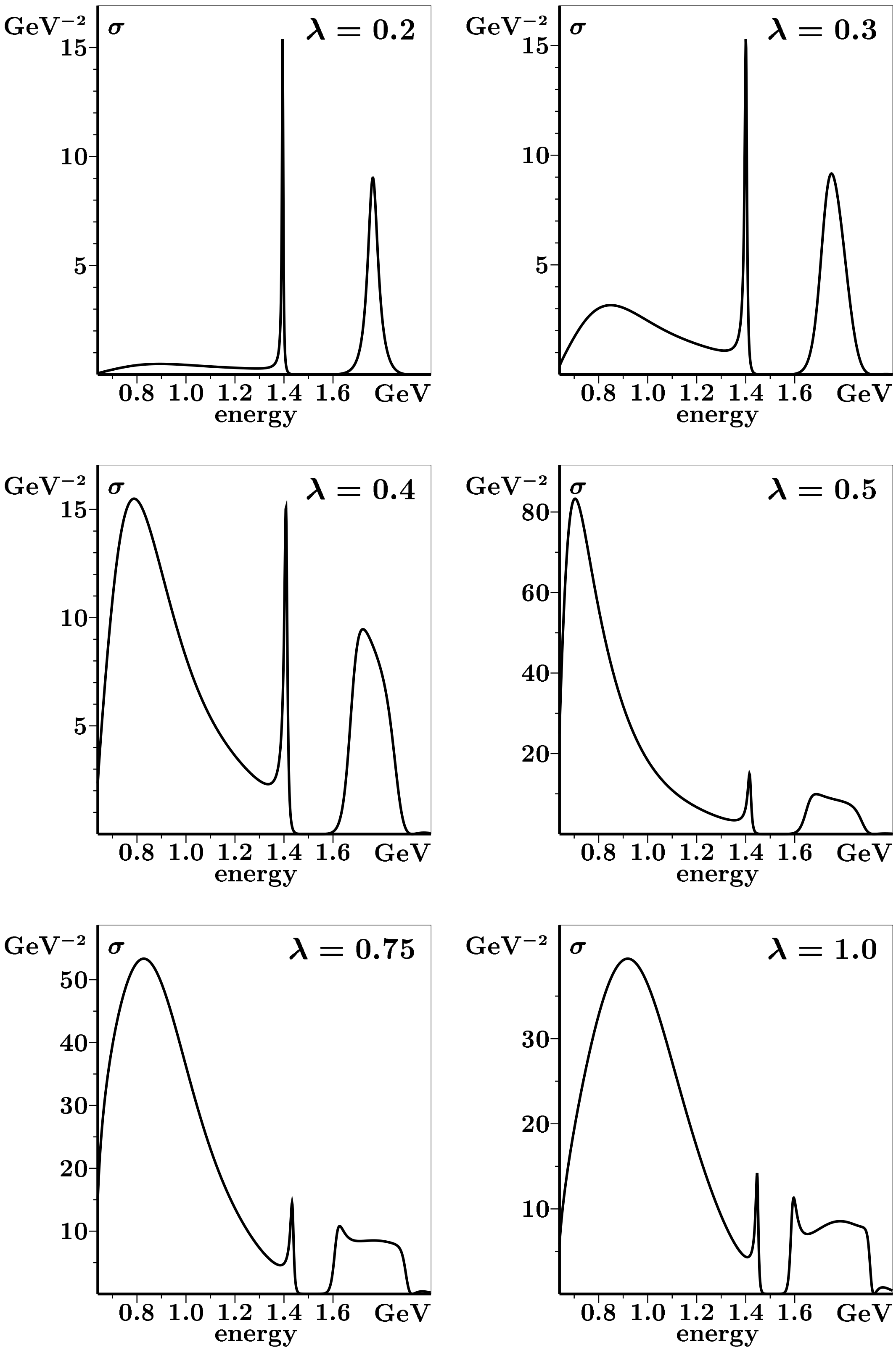
Let us start by studying the elastic scattering in S wave of Kaons and pions for total isospin $I = 1/2$, within a harmonic oscillator model for confinement.

Below we represent the scattering cross section produced by the model, while taking the harmonic oscillator ground state at 1.389 GeV and a level spacing of 380 MeV.



λ is the parameter which describes the intensity of the coupling between the confinement states (harmonic oscillator states in the present case) and the $K\pi$ continuum.

In the following page we study what happens when we only modify λ, nothing else.



GeV^{-2}
σ
$\lambda = 0.2$
15
10
5
0.8 1.0 1.2 1.4 1.6 GeV
energy
GeV^{-2}
σ
$\lambda = 0.3$
15
10
5
0.8 1.0 1.2 1.4 1.6 GeV
energy
GeV^{-2}
σ
$\lambda = 0.4$
15
10
5
0.8 1.0 1.2 1.4 1.6 GeV
energy
GeV^{-2}
σ
$\lambda = 0.5$
80
60
40
20
0.8 1.0 1.2 1.4 1.6 GeV
energy
GeV^{-2}
σ
$\lambda = 0.75$
50
40
30
20
10
0.8 1.0 1.2 1.4 1.6 GeV
energy
GeV^{-2}
σ
$\lambda = 1.0$
30
20
10
0.8 1.0 1.2 1.4 1.6 GeV
energy

$|\psi_f|^2$ = probability meson-meson
$|\psi_c|^2$ = probability quark-antiquark

$$\begin{cases} H_f\,\psi_f(\vec{r}\,) \;+\; V_t\,\psi_c(\vec{r}\,) \;=\; E\,\psi_f(\vec{r}\,) \\ H_c\,\psi_c(\vec{r}\,) \;+\; V_t\,\psi_f(\vec{r}\,) \;=\; E\,\psi_c(\vec{r}\,) \end{cases}$$

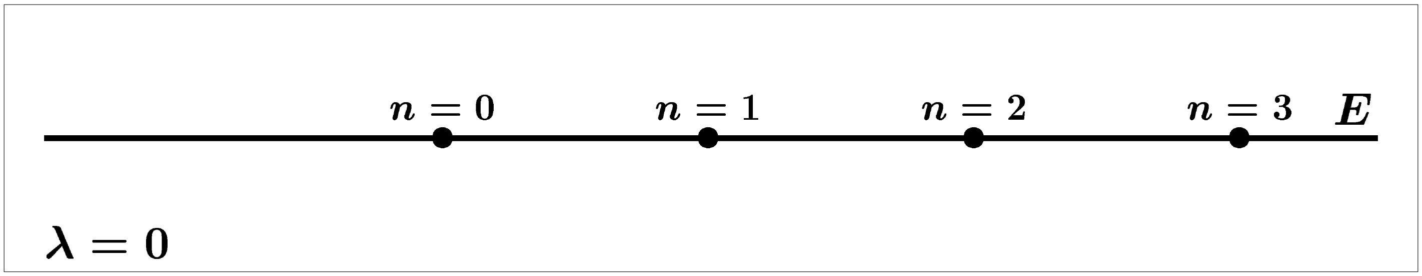
quark-antiquark is unobservable

$$(E - H_f)\;\psi_f(\vec{r}\,) \;=\; V_t\;(E - H_c)^{-1}\;V_t\,\psi_f(\vec{r}\,)$$

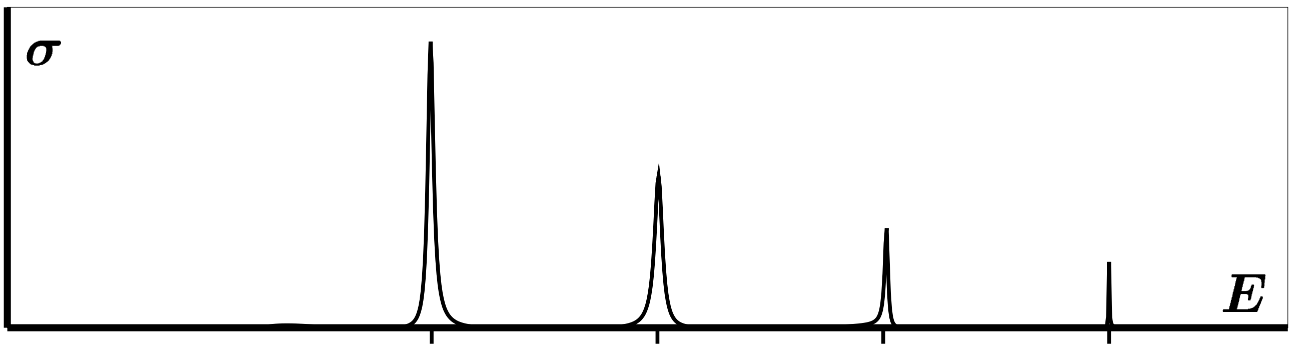
Complete Solution (partial-wave K matrix):

$$K_\ell(p) \;=\; \frac{\pi\lambda^2\mu p \displaystyle\sum_{n=0}^{\infty} \frac{\mathcal{J}^*_{n\ell}(p)\;\mathcal{J}_{n\ell}(p)}{E(p) - E_{n\ell_c}}}{\pi\lambda^2\mu p \displaystyle\sum_{n=0}^{\infty} \frac{\mathcal{J}^*_{n\ell}(p)\;\mathcal{N}_{n\ell}(p)}{E(p) - E_{n\ell_c}} \;-\; 1}$$

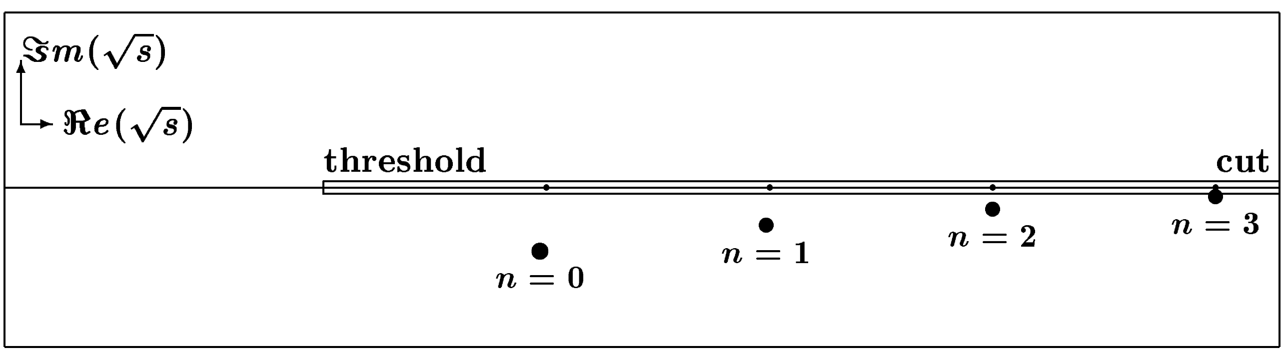
$E_{n\ell_c}$ = radial spectrum quark-antiquark



The spectrum of confinement



Elastic meson-meson scattering (λ small)

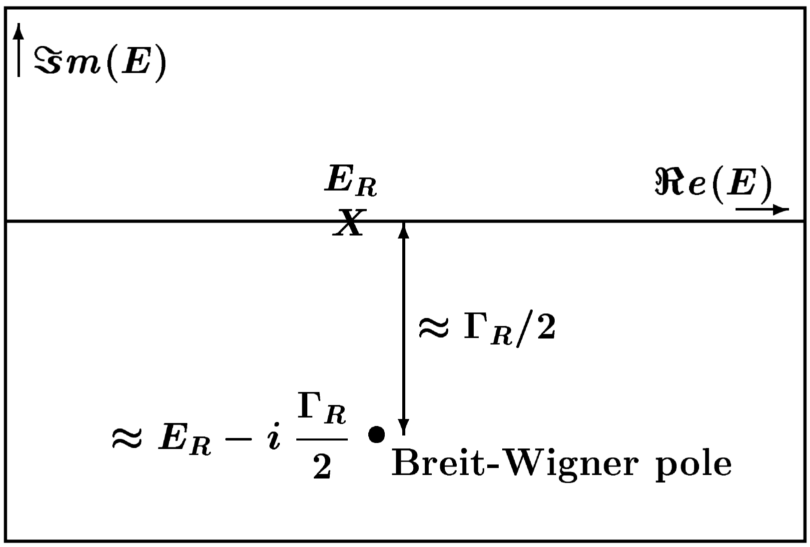


Scattering-matrix poles (λ small)

Near a Breit-Wigner Resonance (λ small)

$$K_\ell(s) \approx \frac{\Gamma_R/2}{E_R - \sqrt{s}}$$

$E_R \approx$ central resonance mass
$\Gamma_R \approx$ resonance width



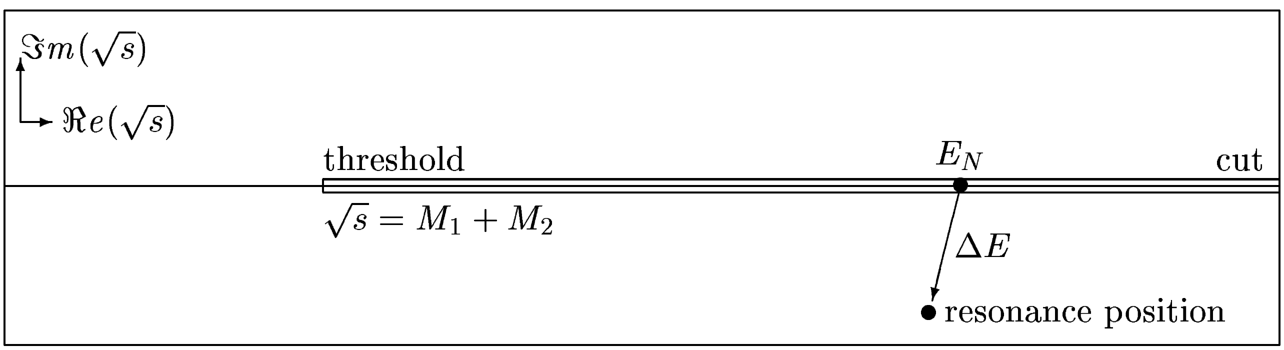
complex $E(p)$ plane

Our resonance expression:

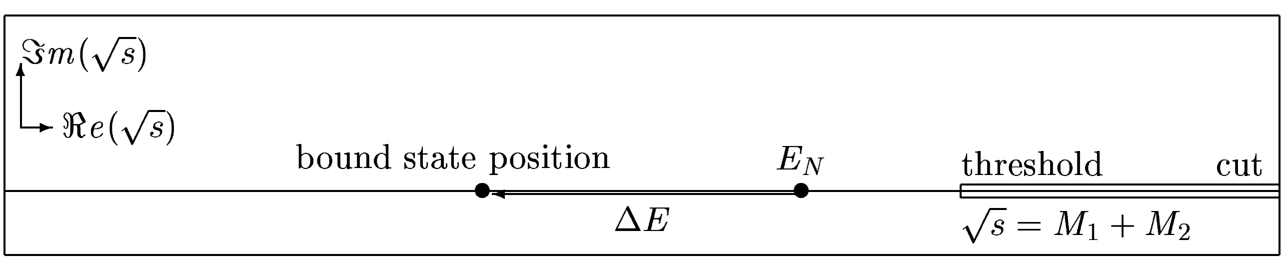
$$K_\ell(p) \;=\; \frac{\pi\lambda^2\mu p \displaystyle\sum_{n=0}^{\infty} \frac{\mathcal{J}^*_{n\ell}(p)\;\mathcal{J}_{n\ell}(p)}{E(p) - E_n}}{\pi\lambda^2\mu p \displaystyle\sum_{n=0}^{\infty} \frac{\mathcal{J}^*_{n\ell}(p)\;\mathcal{N}_{n\ell}(p)}{E(p) - E_n} \;-\; 1}$$

There are two cases:

1. E_N above threshold



2. E_N below threshold



Approximation in our expression for a better contact with the physics

$$K_\ell(p) \approx \frac{2\lambda^2\ \mu\ pa\ j_\ell^2(pa) \sum\limits_{n=0}^{\infty} \frac{|\mathcal{F}_n(a)|^2}{E(p)-E_n}}{2\lambda^2\ \mu\ pa\ j_\ell(pa)\ n_\ell(pa) \sum\limits_{n=0}^{\infty} \frac{|\mathcal{F}_n(a)|^2}{E(p)-E_n} - 1}$$

λ = coupling constant
p = relative meson-meson linear momentum
$E(p)$ = total invariant meson-meson mass
E_n = n-th level of the confinement spectrum
μ = reduced meson-meson mass
j_ℓ = spherical Bessel function
n_ℓ = spherical Neumann function
$\mathcal{F}_n$ = quark-antiquark confinement wave function
a = $q\bar{q}$ separation distance (≈ 0.5 fm)

and a further approximation

$$\lambda^2 \sum_{n=0}^{\infty} \frac{|\mathcal{F}_n(a)|^2}{E-E_n} \approx \lambda^2 \left(\sum_{n=0}^{N} \frac{B_n}{E-E_n} - 1 \right)$$

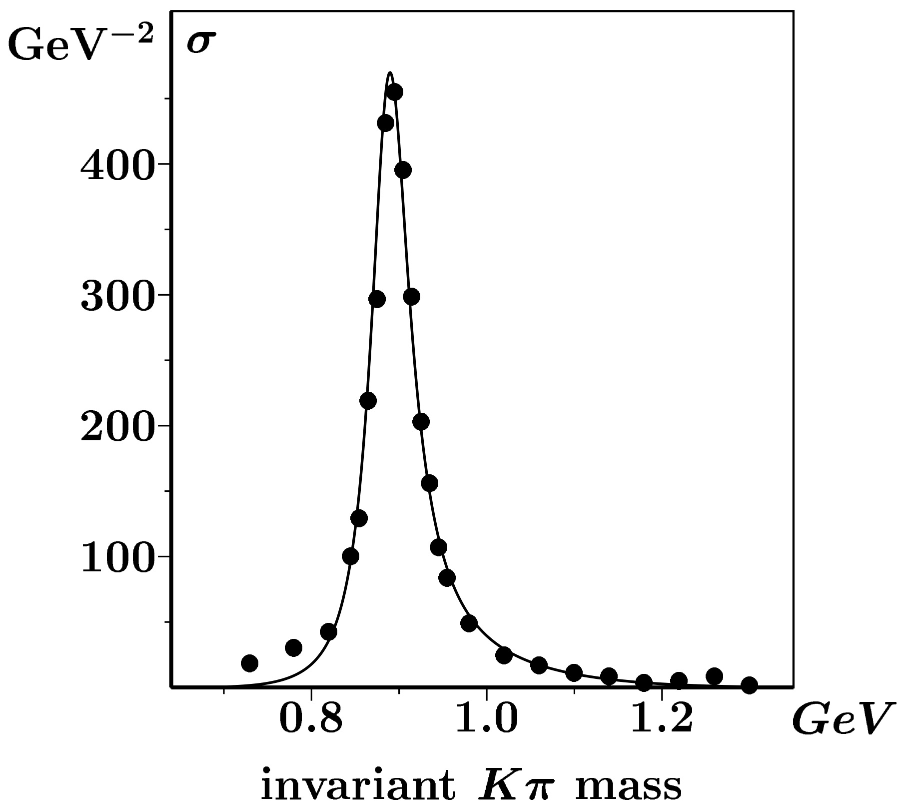
with a redefinition of λ

$K\pi$

Elastic $I = \frac{1}{2}$ P-wave scattering

$\lambda = 0.75\ \text{GeV}^{-3/2}$ and $a = 5\ \text{GeV}^{-1}$

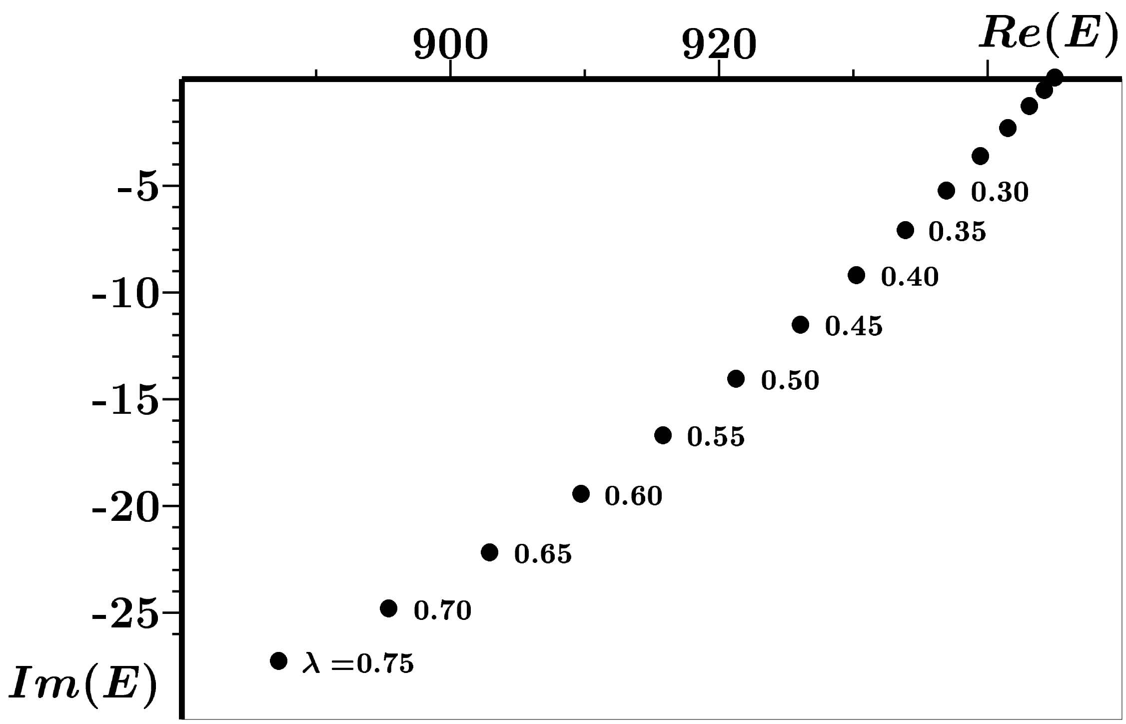
$$\sum_{n=0}^{\infty} \frac{|\mathcal{F}_n(a)|^2}{E - E_n} \approx \left(\frac{0.5}{E - 0.945} - 1 \right) \text{GeV}^2$$



Complex-energy singularities of the S-matrix as function of λ

The point on the real axis corresponds to the bare state ($\lambda = 0$) at 945 MeV

Units are in MeV



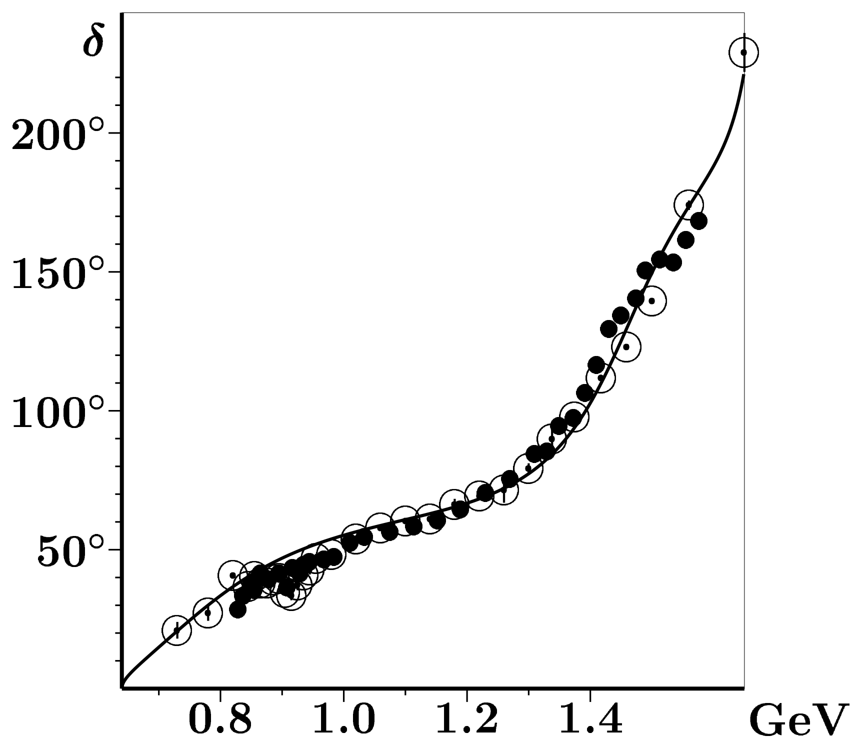
Pole at 0.887 - 0.027i GeV

$K\pi$

Elastic $I = \frac{1}{2}$ S-wave scattering

$\lambda = 0.75$ GeV$^{-3/2}$ and $a = 3.2$ GeV^{-1}

$$\left(\frac{1.0}{E - 1.31} + \frac{0.2}{E - 1.69} - 1 \right) \text{GeV}^2$$

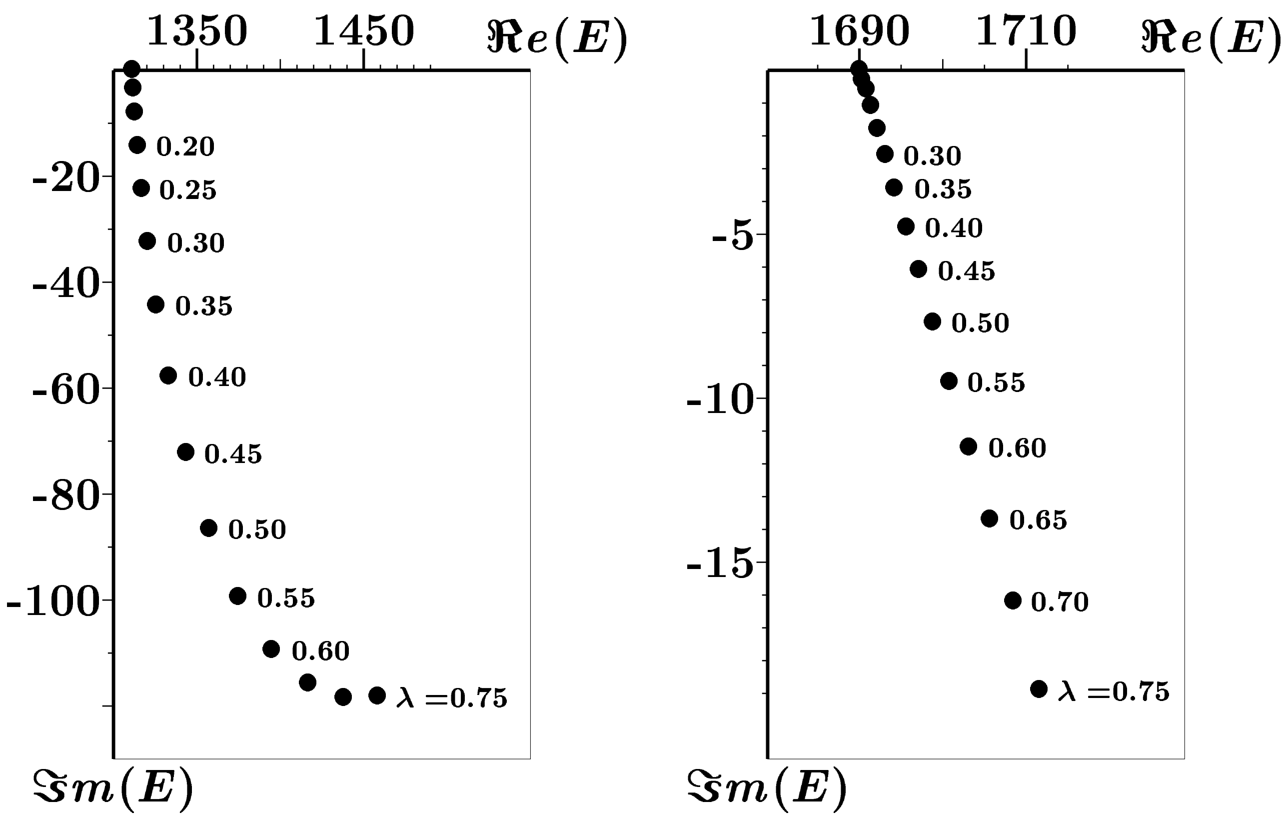


total CM energy

Complex-energy singularities of the S-matrix as function of λ

The points on the real axis correspond to the bare states ($\lambda = 0$)

Units are in MeV



Notice nonperturbative behaviour of lower singularity

and a singularity at

713 - 227i MeV

in

E. van Beveren, T. A. Rijken, K. Metzger,
C. Dullemond, G. Rupp, and J. E. Ribeiro
Zeitschrift für Physik C30, 615 (1986)

found at

727 - 263i MeV

many more channels

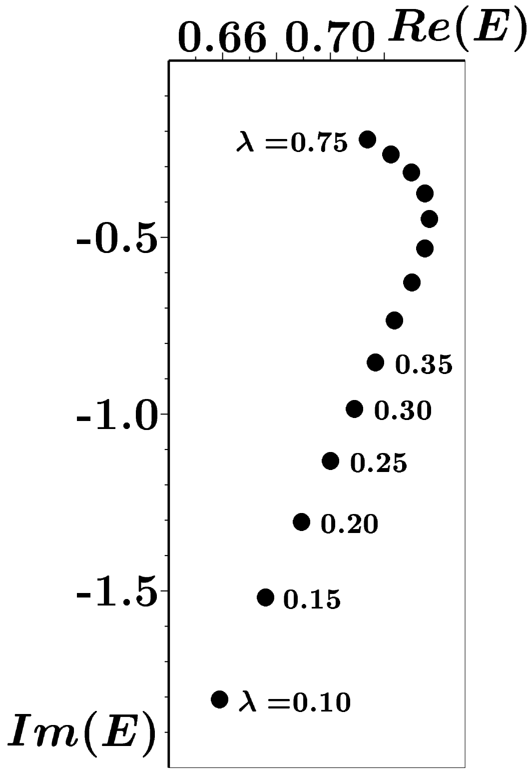
full transition potential

harmonic oscillator confinement

no free parameters

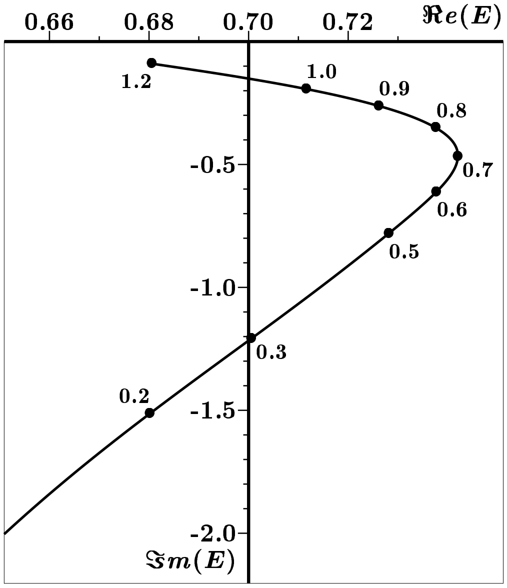
Complex-energy singularities of the S-matrix as function of λ

Singularity disappears in background for $\lambda = 0$

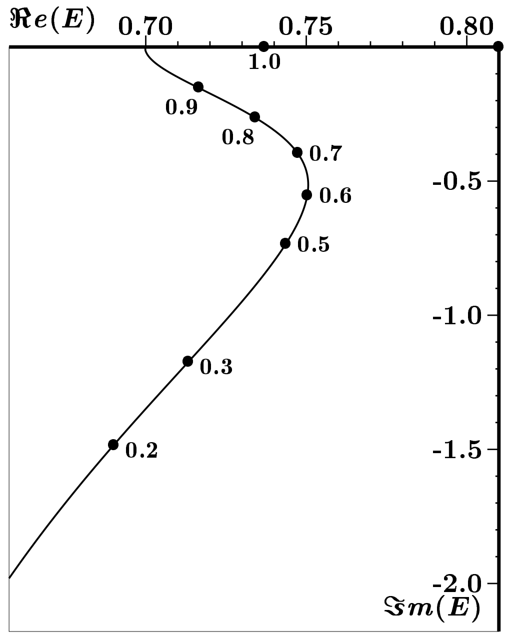


Units are in GeV

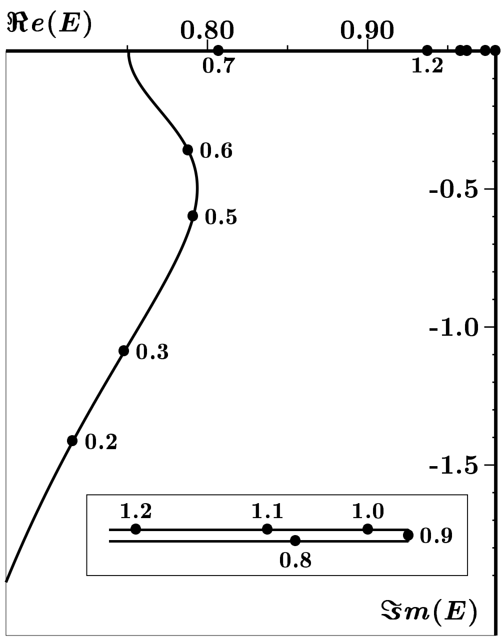
Study of the extra pole threshold dependence



(a)



(b)



(c)

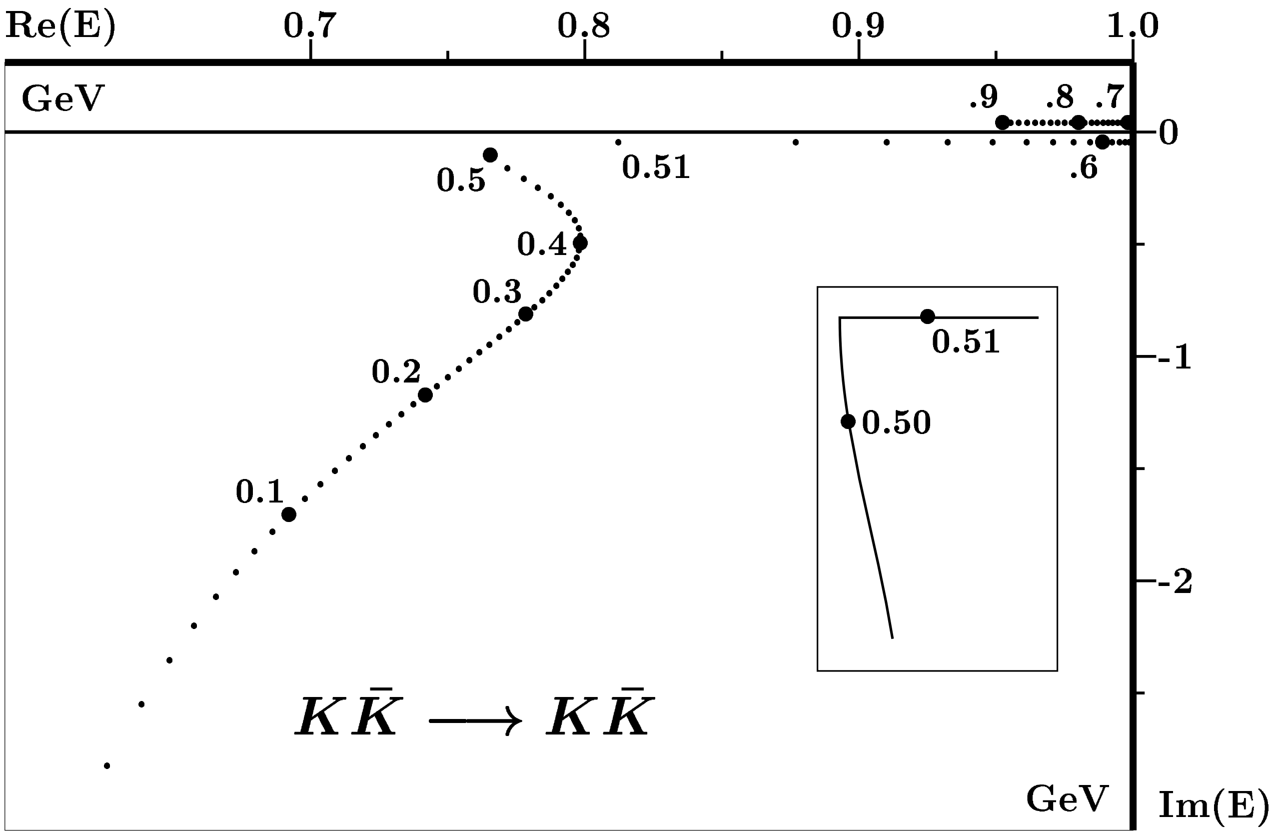
figure	threshold
a	0.70 GeV
b	0.81 GeV
c	0.99 GeV

$\lambda = 1.0$ here, corresponds to $\lambda = 0.75$ in the other figures

KK, elastic $I = 1$ S-wave scattering

$\lambda = 0.75$ GeV$^{-3/2}$ and $a = 3.2$ GeV^{-1}

$$\left(\frac{1.0}{E-1.21} + \frac{0.2}{E-1.59} - 1\right) \text{GeV}^2$$



$$\text{But } KK \leftrightarrow \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \leftrightarrow \eta\pi$$

This gives a width to the $a_0(980)$
We find the pole at $962 - i28$ MeV.

We find a nonet

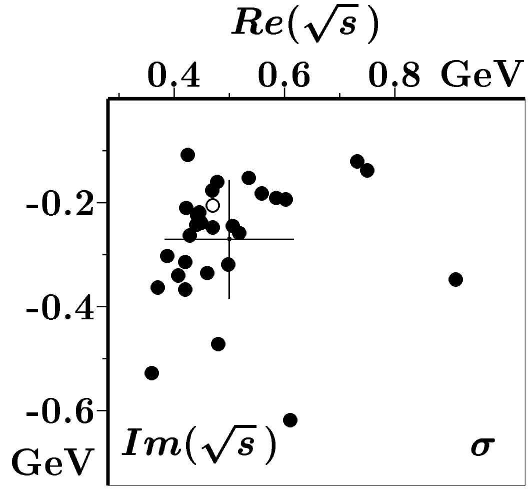
of extra poles

in S-wave scattering

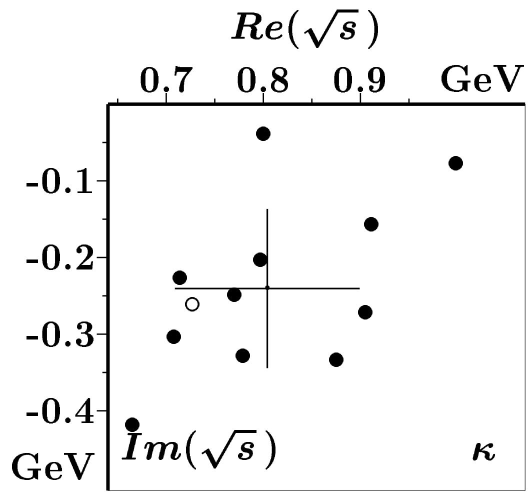
Isospin	pole position (MeV)
$I = 1$	968-28i
$I = \frac{1}{2}$	727-263i
$I = 0$	470-208i and 994-17i

forms

THE nonet of
the lowest lying singularities
of the scattering matrix
for $J^P = 0^+$ states



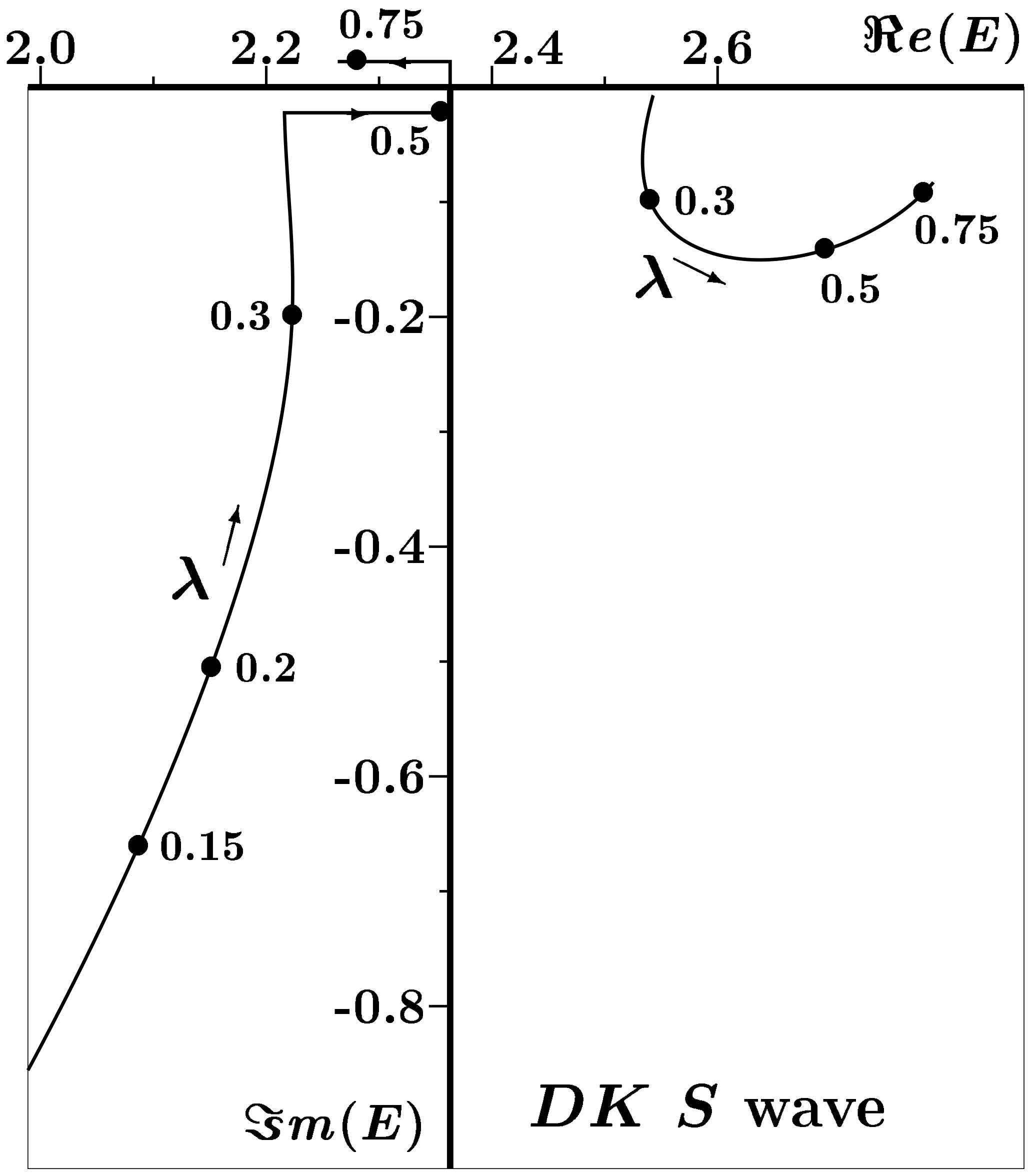
World average for sigma equals $(500 \pm 117) - i(271 \pm 114)$ MeV.



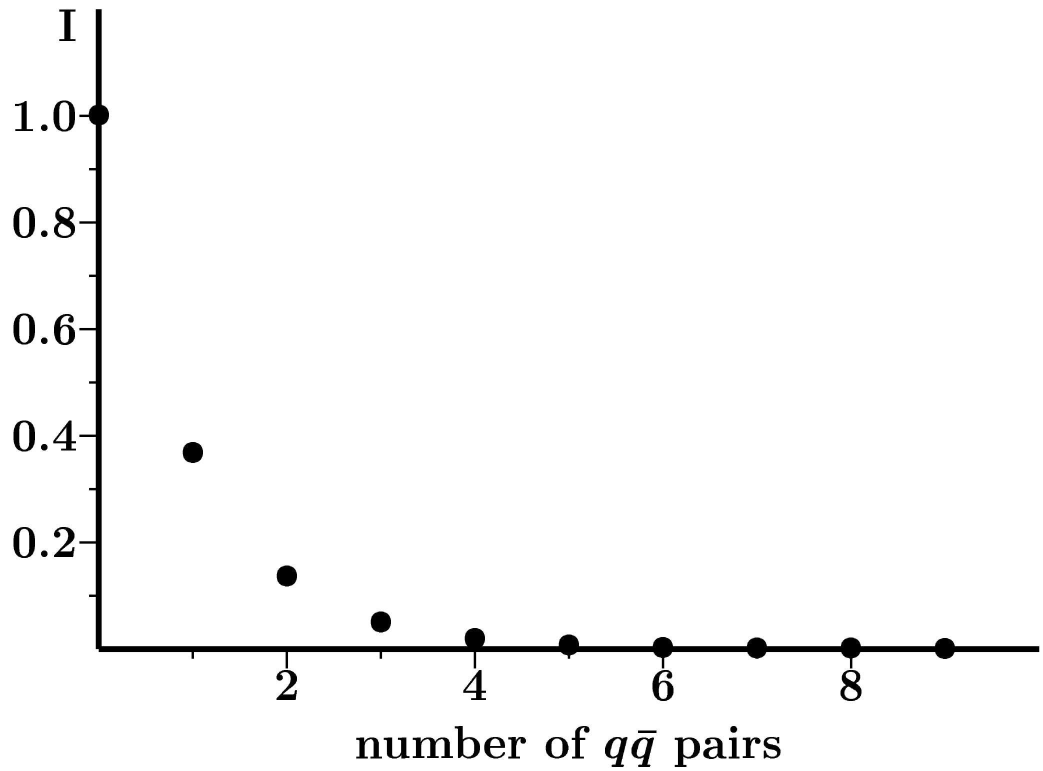
World average for kappa equals $(804 \pm 95) - i(241 \pm 104)$ MeV.

$E_0 = 2545$ MeV and $E_1 = 2925$ MeV

the other parameters unaltered

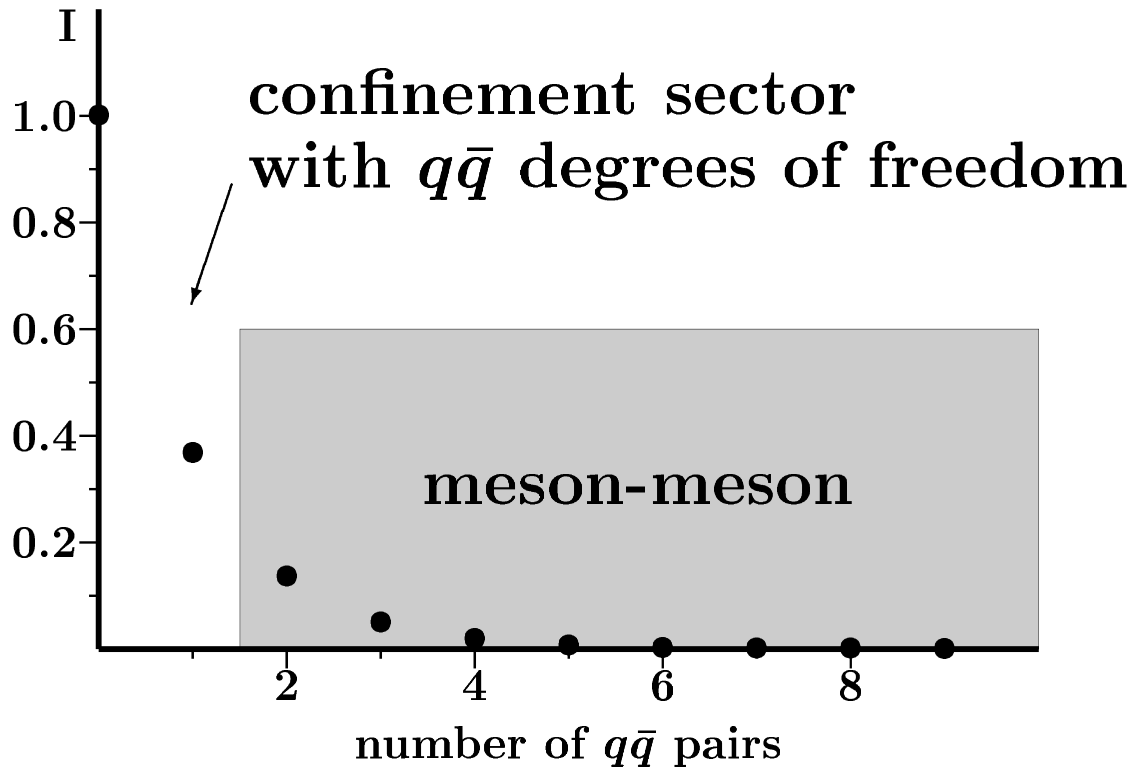


We find the $D^*_{sJ}(2317)$.



The importance I of contribution to the dynamics of mesonic states for different configurations of $q\bar{q}$ pairs, as a function of the number of $q\bar{q}$ pairs.

# $q\bar{q}$	I
0	**just glue, very important for confinement and for the effective quark masses**
1	**gives the degrees of freedom to mesonic systems**
2	**mediates the coupling to two-meson systems**
3	**mediates the coupling to three-meson systems**



One has then

1. **confinement spectrum**

2. **deformed by communication to two-meson sector**

   - **mass shifts**
   - **resonance widths**
   - **extra resonances/bound states**

CONCLUSION(S)

The expression

$$K_\ell(p) \approx \frac{2\lambda^2 \mu\, pa\, j_\ell^2(pa) \displaystyle\sum_{n=0}^{\infty} \frac{|\mathcal{F}_n(a)|^2}{E(p)-E_n}}{2\lambda^2 \mu\, pa\, j_\ell(pa)\, n_\ell(pa) \displaystyle\sum_{n=0}^{\infty} \frac{|\mathcal{F}_n(a)|^2}{E(p)-E_n} - 1}$$

seems a good approximation for data analysis.

Full off-shell T matrix:

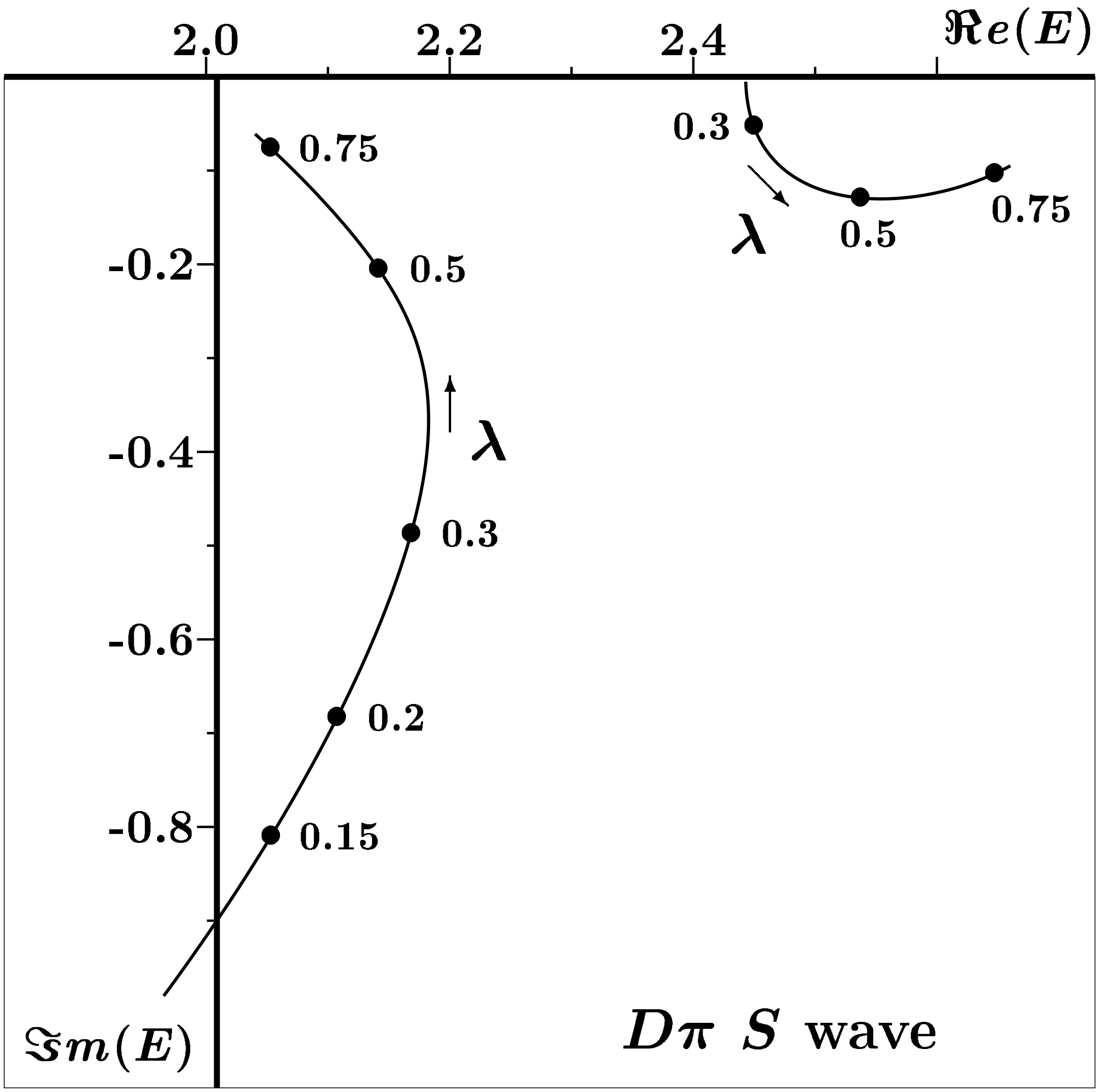
- hep-ph/0304105 (δ-shell for V_t)
- hep-ph/0306155 (in the appendix, more general)

Many-channel analysis of light scalar mesons:

- Zeitschrift für Physik C30, 615 (1986)
(postscript version available through Spires)

$E_0 = 2443$ MeV and $E_1 = 2823$ MeV

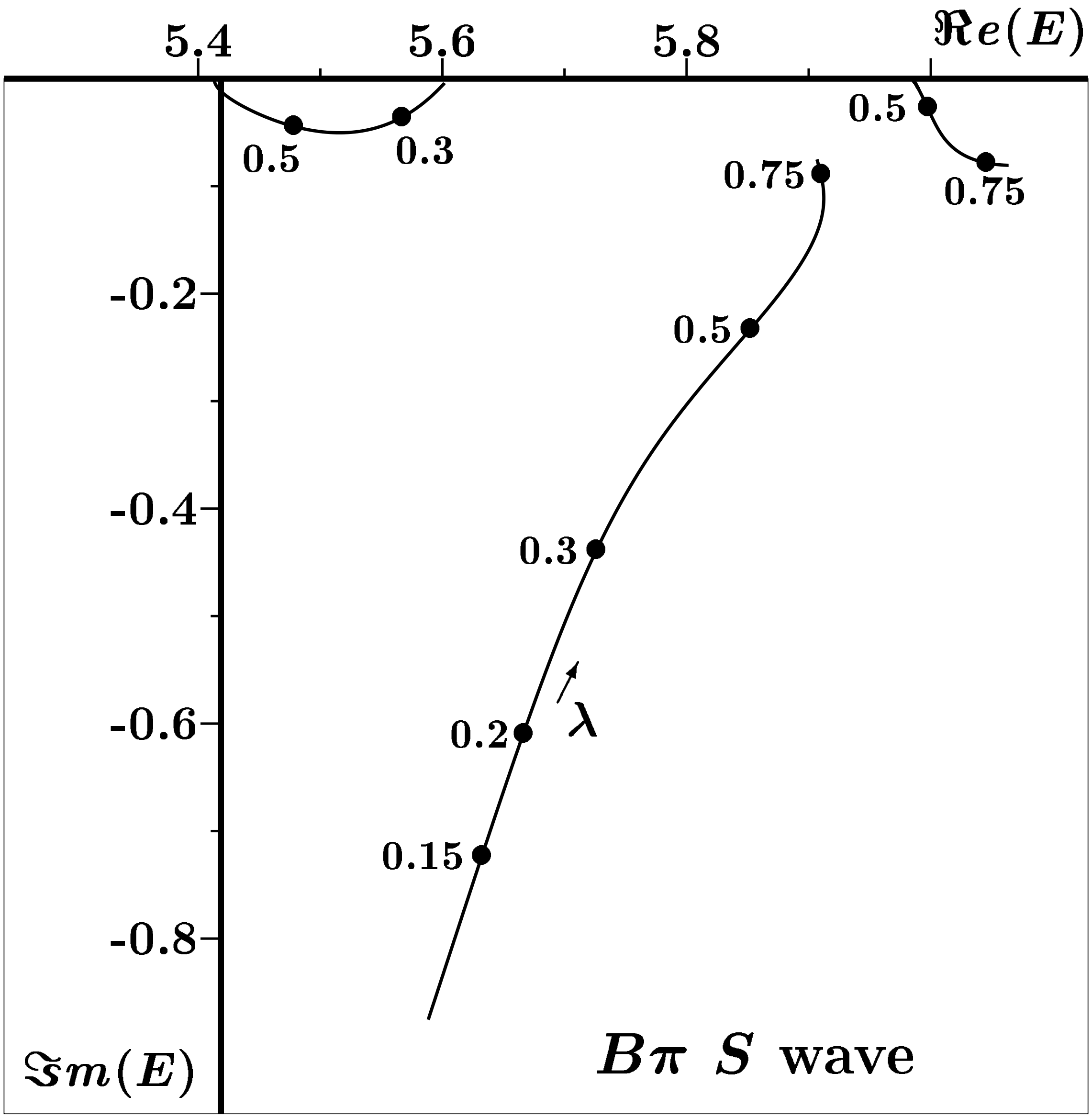
the other parameters unaltered



We find the D_0^*(2100-2300) and D_0^*(2640).

$E_0 = 5605$ MeV and $E_1 = 5985$ MeV

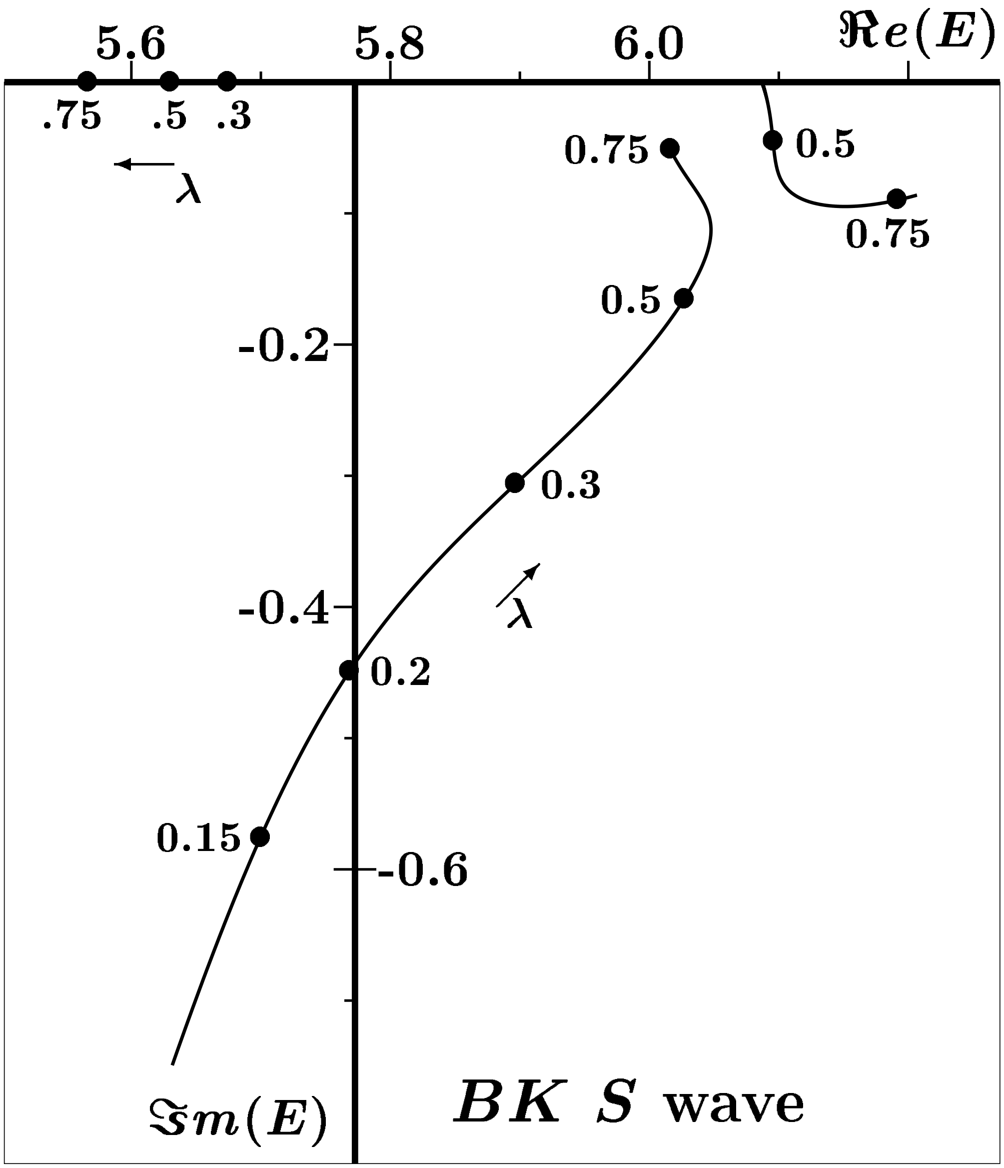
the other parameters unaltered



We find the $B_0^*(5400\text{-}5450)$ just at threshold, $B_0^*(5900)$ and $B_0^*(6050)$.

$E_0 = 5707$ MeV and $E_1 = 6087$ MeV

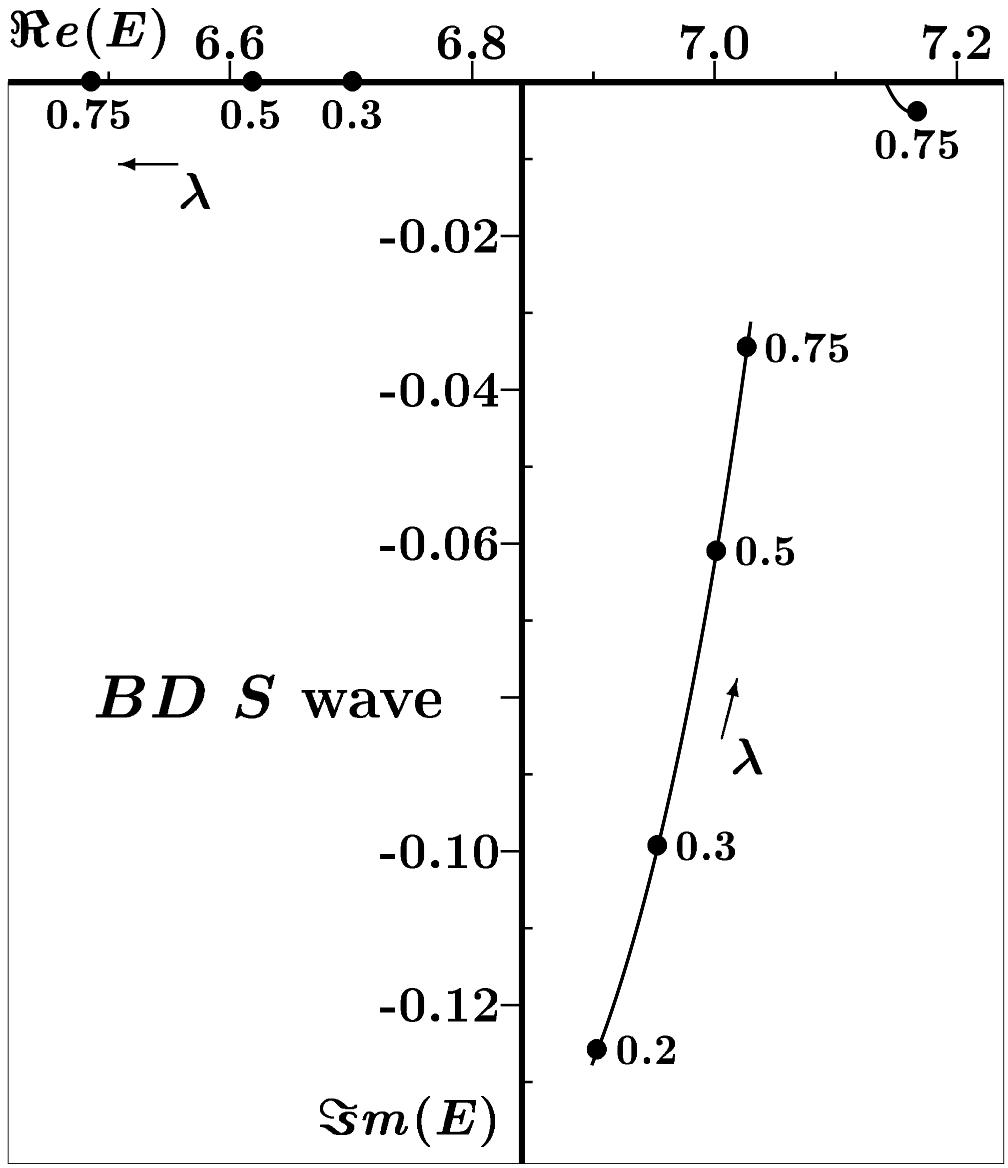
the other parameters unaltered



We find the $B^*_{s0}(5570)$ below threshold, $B^*_{s0}(6000)$ and $B^*_{s0}(6200)$.

$E_0 = 6761$ MeV and $E_1 = 7141$ MeV

the other parameters unaltered



We find the $B^*_{c0}(6500)$ below threshold, $B^*_{c0}(7000)$ and $B^*_{c0}(7170)$.