



Final state interaction effects on the reactions producing meson with a light nuclei

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Introduction

- The intermediate energy physics needs information on the baryon resonances.
- Effective field theories replace the QCD at these energies.
- The meson-nucleon interaction, mNN^* coupling constants are essential ingredients for such theories.
- Apart from all this, the meson-nucleus/nucleon physics seem to also address fundamental problems like charge symmetry breaking, OZI rule etc.
- Interesting physics like mesic states can be explored

- A study of meson producing reactions is a good alternative to obtain information on the meson-nucleus and hence meson nucleon interaction.
- In case of the neutral mesons, these reactions are the only way to get such information.
- The reactions which produce mesons with light nuclei are especially interesting since the final states of such systems can be solved almost exactly in terms of few body equations.

The p d \rightarrow ^3He η reaction close to threshold.

- Data available very close to the threshold (upto 10 MeV above threshold).^{1,2}
- An effort to explain the data has been made by many theoretical groups^{3,4,5} using the two step model and contribution from the any other direct mechanism was found to be negligible⁴.
- We studied the p d \rightarrow ^3He η reaction close to threshold with the aim to study the effect of the ^3He - η final state interaction on the reaction cross-section.
- We used the two step model for the production mechanism.

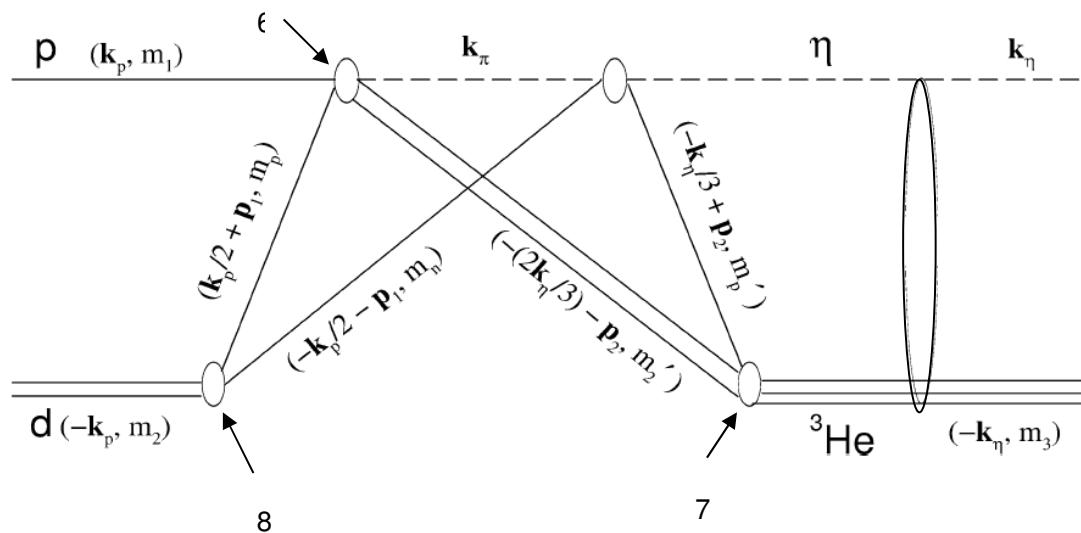
¹B. Mayer et al., Phys. Rev. C 53 (1996) 2068. ²J. Berger et al., Phys. Rev. Lett. 61 (1988) 919.

³G. Fäldt, C. Wilkin, Nucl. Phys. A 587 (1995) 769. ⁴J.M. Laget, J.F. Lecolley, Phys. Rev. Lett. 61 (1988) 2069.

⁵L.A. Kondratyuk, A.V. Lado, Yu.N. Uzikov, Phys. At. Nucl. 58 (1995) 473

Study of the η -nucleus interaction in the $pd \rightarrow {}^3\text{He} \eta$ reaction near threshold

www.elsevier.com/locate/npe

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$$\langle |T_{pd \rightarrow {}^3\text{He}\eta}| \rangle = i \int \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3} \sum_{\text{int } m's} \langle pn|d\rangle \langle \pi^+ d| T_{pp \rightarrow \pi+d} |pp\rangle \times \frac{1}{(k_\pi^2 - m_\pi^2 + i\epsilon)} \langle \eta p| T_{\pi N \rightarrow \eta p} |\pi^+ n\rangle \langle {}^3\text{He}|pd\rangle$$

⁶R.A. Arndt, I. Strakovsky, R.L. Workman, D.A. Bugg, Phys. Rev. C 48 (1993) 1926.

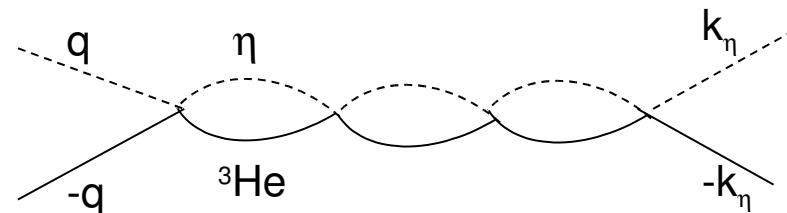
⁷J.F. Germond, C. Wilkin, J. Phys. G 14 (1988) 181.

⁸M. Lacombe, B. Loiseau, R. Vinh Mau, J. Côté, P. Pirés, R. de Tourreil, Phys. Lett. B 101 (1981) 139.

The transition matrix for the reaction $pd \rightarrow {}^3\text{He} \eta$,

$$T = \langle \Psi_{\eta {}^3\text{He}}^- (\vec{k}_\eta); m_3 | T_{pd \rightarrow {}^3\text{He} \eta} | \vec{k}_p; m_1 m_2 \rangle,$$

$$\langle \Psi_{\eta {}^3\text{He}}^- | = \langle \vec{k}_\eta | + \int \frac{d\vec{q}}{(2\pi)^3} \frac{\langle \vec{k}_\eta | T_{\eta {}^3\text{He}} | \vec{q} \rangle}{E(k_\eta) - E(q) + i\epsilon} \langle \vec{q} |,$$



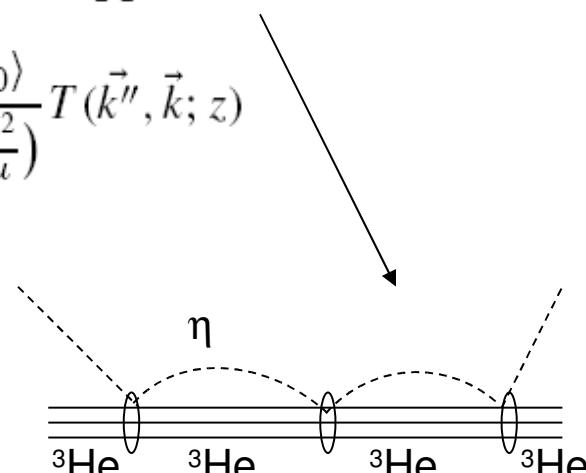
$$T = \langle \vec{k}_\eta; m_3 | T_{pd \rightarrow {}^3\text{He} \eta} | \vec{k}_p; m_1 m_2 \rangle + \sum_{m'_3} \int \frac{d\vec{q}}{(2\pi)^3} \frac{\langle \vec{k}_\eta; m_3 | T_{\eta {}^3\text{He}} | \vec{q}; m'_3 \rangle}{E(k_\eta) - E(q) + i\epsilon} \langle \vec{q}; m'_3 | T_{pd \rightarrow {}^3\text{He} \eta} | \vec{k}_p; m_1 m_2 \rangle.$$

The $\eta {}^3\text{He}$ T-matrix within a Finite Rank Approximation

$$T(\vec{k}', \vec{k}; z) = \langle \vec{k}'; \psi_0 | T^0(z) | \vec{k}; \psi_0 \rangle + \varepsilon \int \frac{d\vec{k}''}{(2\pi)^3} \frac{\langle \vec{k}'; \psi_0 | T^0(z) | \vec{k}''; \psi_0 \rangle}{(z - \frac{k''^2}{2\mu})(z - \varepsilon - \frac{k''^2}{2\mu})} T(\vec{k}'', \vec{k}; z)$$

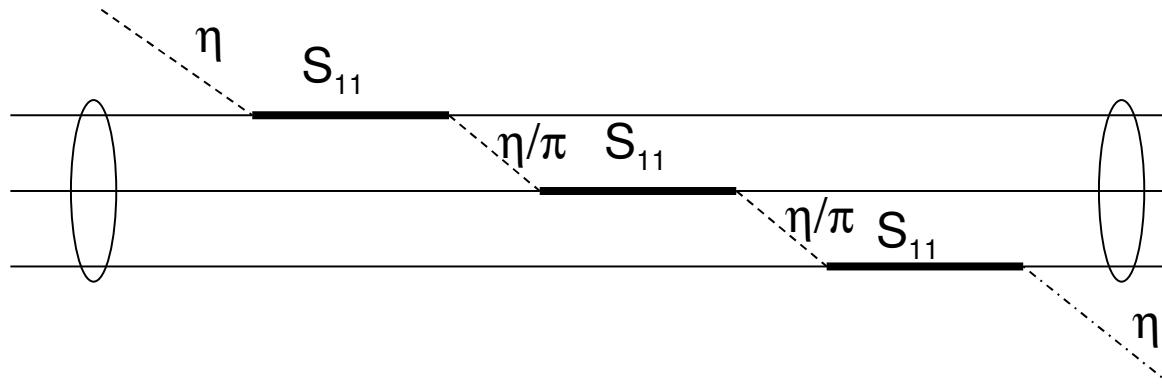
$$\langle \vec{k}'; \psi_0 | T^0(z) | \vec{k}; \psi_0 \rangle = \int d\vec{r} |\psi_0(\vec{r})|^2 T^0(\vec{k}', \vec{k}; \vec{r}; z).$$

$$T^0(\vec{k}', \vec{k}; \vec{r}; z) = \sum_{i=1}^A T_i^0(\vec{k}', \vec{k}; \vec{r}_i; z).$$



$$T_i^0(\vec{k}', \vec{k}; \vec{r}_i; z) = t_i(\vec{k}', \vec{k}; \vec{r}_i; z) + \int \frac{d\vec{k}''}{(2\pi)^3} \frac{t_i(\vec{k}', \vec{k}''; \vec{r}_i; z)}{z - \frac{\vec{k}'^2}{2\mu}} \sum_{j \neq i} T_j^0(\vec{k}'', \vec{k}; \vec{r}_j; z).$$

$$t_i(\vec{k}', \vec{k}; \vec{r}_i; z) = t_{\eta N}(\vec{k}', \vec{k}; z) \exp[i(\vec{k} - \vec{k}') \cdot \vec{r}_i]$$



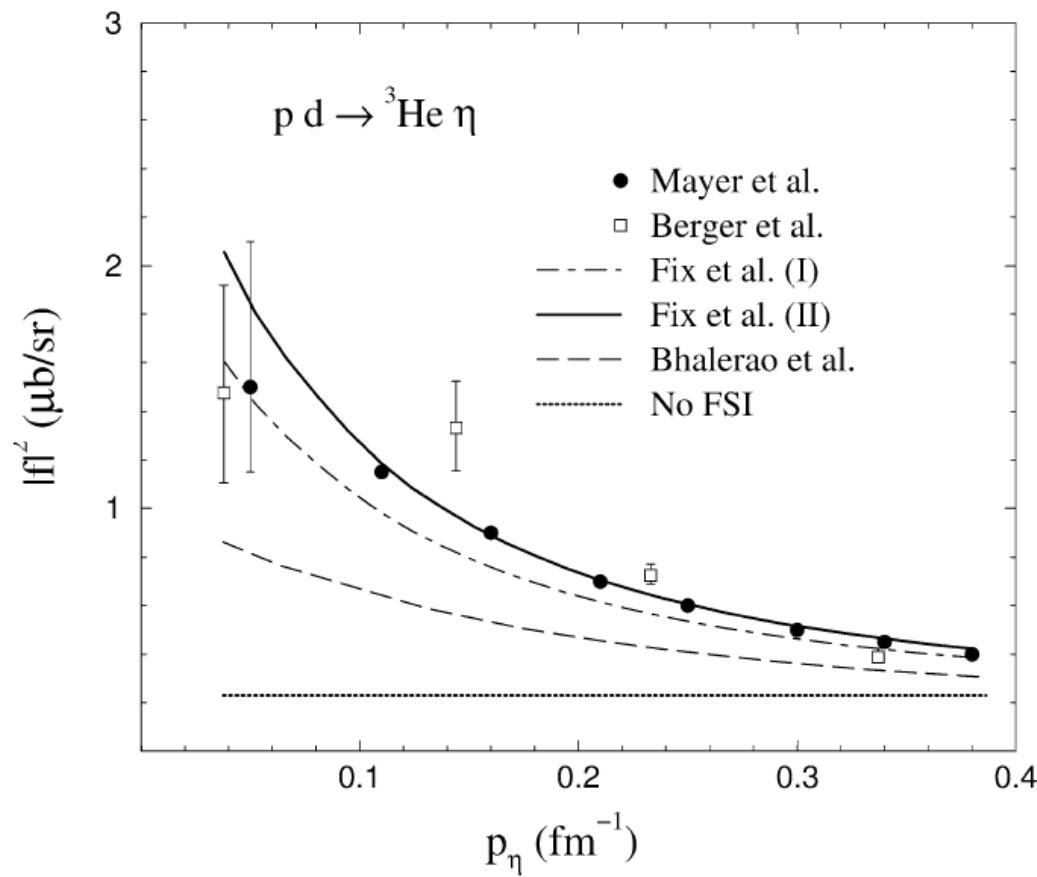
$$^{9,10,11} t_{\eta N \rightarrow \eta N}(k', k; z) = \frac{g_{N^*}\beta^2}{(k'^2 + \beta^2)} \tau_{N^*}(z) \frac{g_{N^*}\beta^2}{(k^2 + \beta^2)}$$

$$\tau_{N^*}(z) = (z - M_0 - \Sigma_\pi(z) - \Sigma_\eta(z) + i\epsilon)^{-1}$$

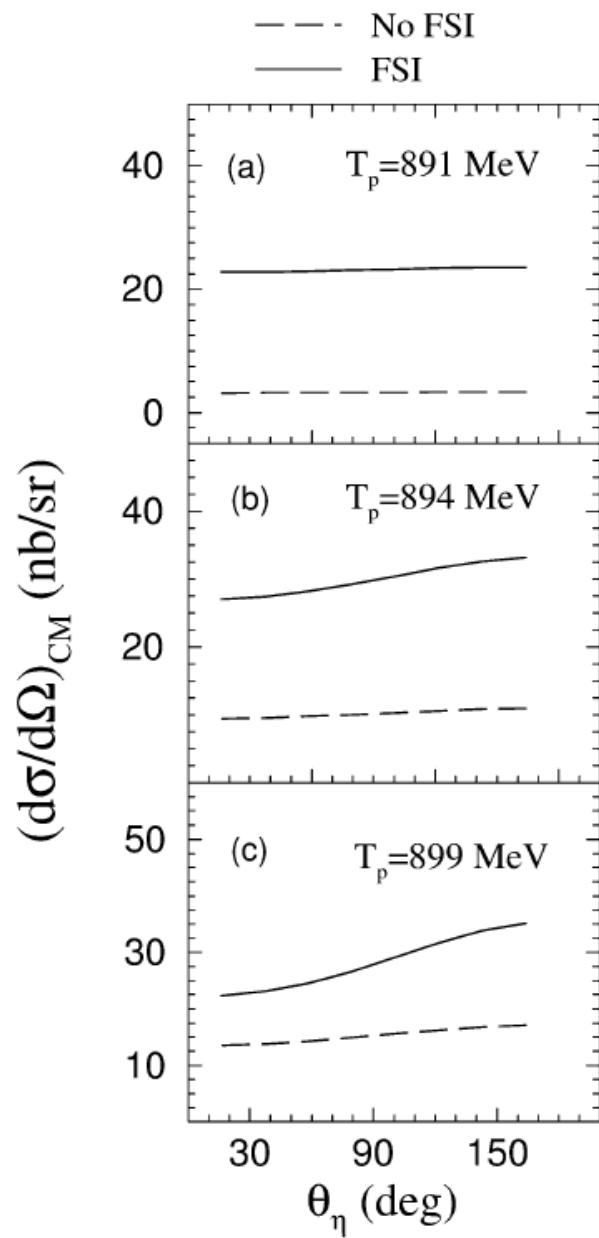
⁹A. Fix, H. Arenhövel, Nucl. Phys A 697 (2002) 277.

¹⁰H. Garcilazo, M.T. Peña, Phys. Rev. C 63 (2001) 021001.

¹¹R.S. Bhalerao, L.C. Liu, Phys. Rev. Lett. 54 (1985) 865.



The square of the $pd \rightarrow {}^3\text{He} \eta$ amplitude



High energy data

- Data available from Uppsala at four beam energies ranging from 930-1100 MeV¹².
- The angular distributions at these energies, which are 90 to 200 MeV energies above threshold, are peaking in the forward zone.
- However, an attempt¹³ to describe these differential cross sections within the two step model using monte carlo decription found peaking at the backward angles
- Another attempt to describe the same data from theoretical point of view¹⁴ found agreement with the data but with arbitrary and unjustified approximations like restricting the intermediate pion to 0 deg.

¹²R. Bilger *et al.*, Phys. Rev. C **65**, 044608 (2002).

¹³J. Złomańczuk *et al.*, Acta Phys. Pol. B **33**, 883 (2002).

¹⁴M. Stenmark, Phys. Rev. C **67**, 034906 (2003).

PHYSICAL REVIEW C **68**, 064610 (2003)

Three-body mechanism of η production

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The $\pi N \rightarrow \eta p$ reaction¹⁵

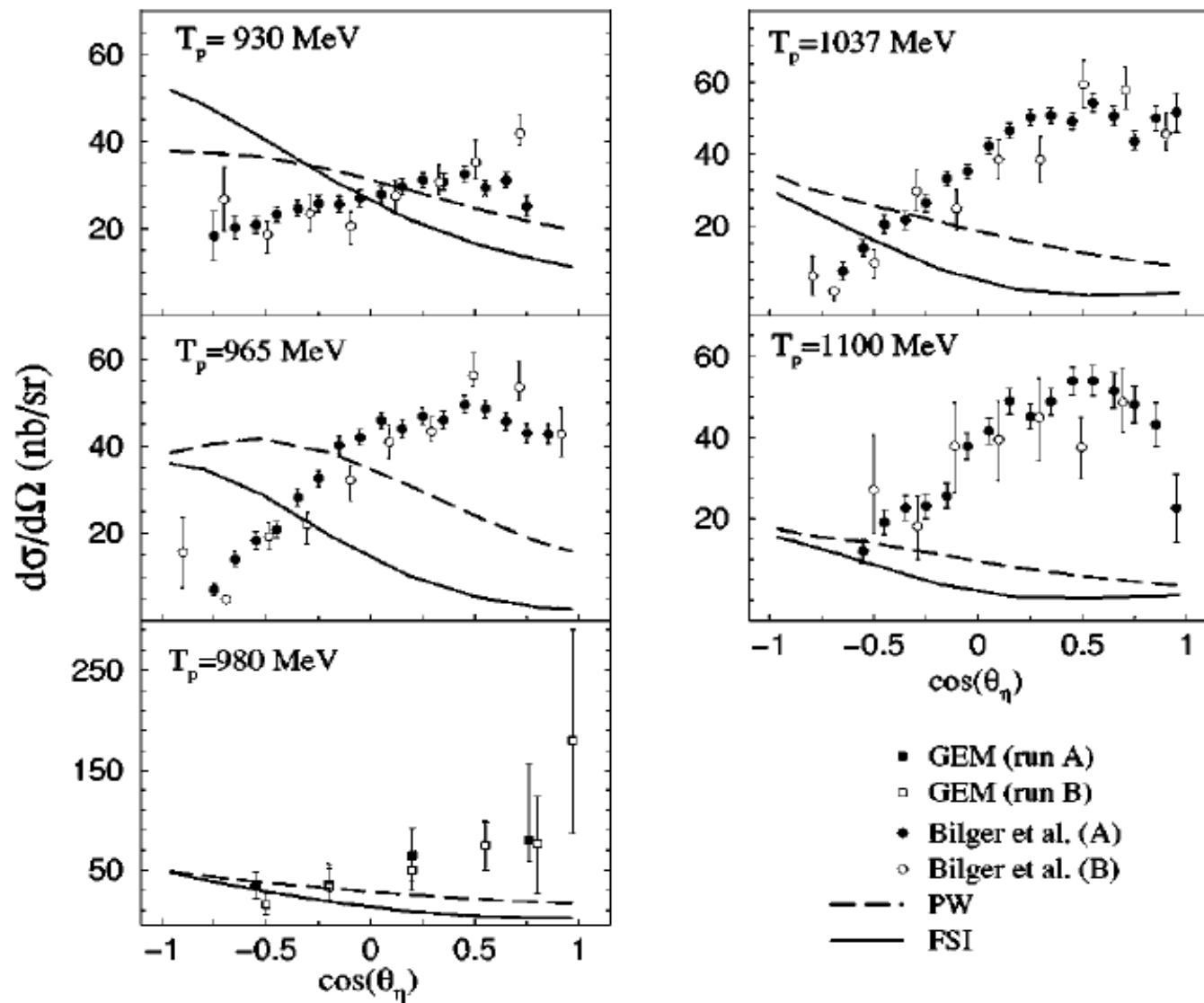
single resonance, coupled channel model

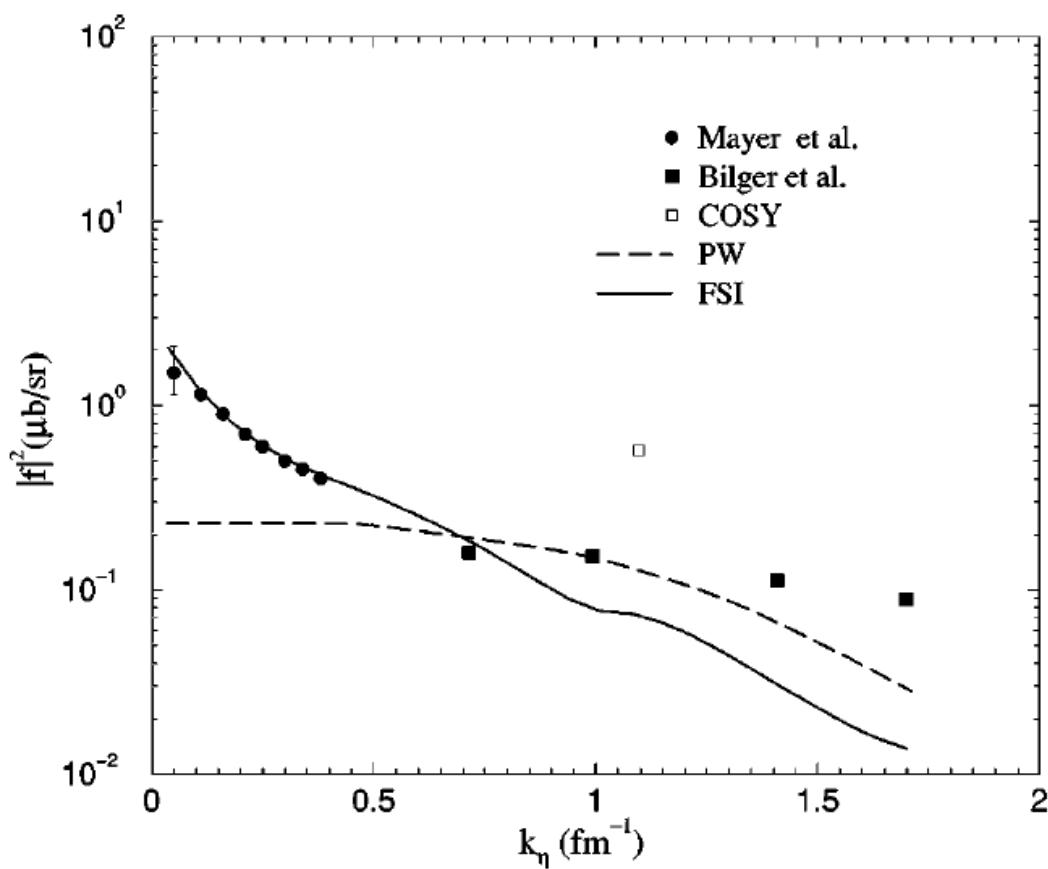
πN , ηN , and $\pi\pi N$

&

inclusion of the S_{11} , P_{11} , and D_{13} resonances.

¹⁵M. Batinić *et al.*, Phys. Rev. C **51**, 2310 (1995)





The $p\ d \rightarrow pd\ \eta$ reaction

- Experimental data available from Uppsala (Wasa-Promice collaboration) at four beam energies ranging from ~ 930 MeV to 1100 MeV.

Interesting features of the data:

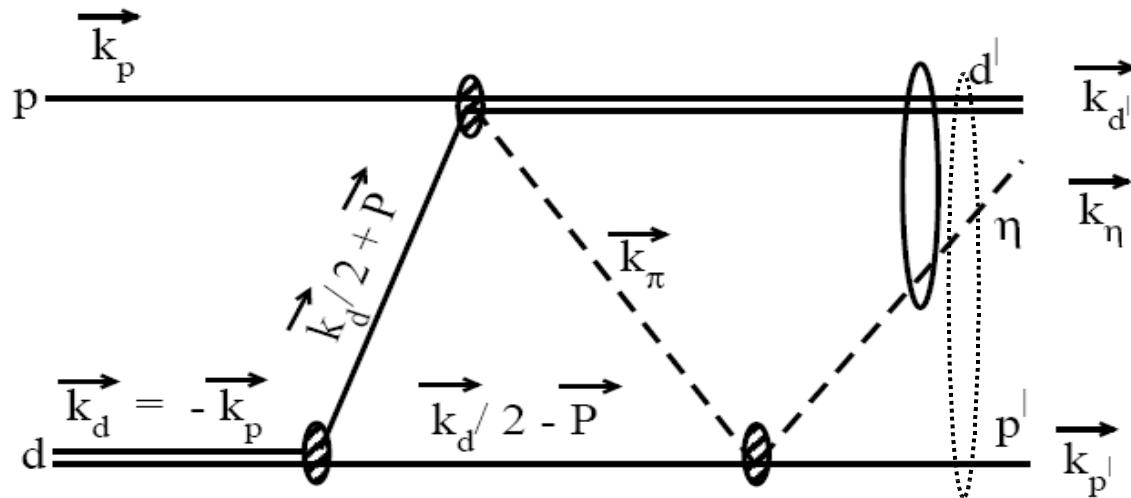
- The invariant mass distributions for the η -p, η -d and p-d systems at the beam energy of 1032 MeV.
- Similar to the $p\ d \rightarrow {}^3\text{He}\ \eta$ reaction, the η -d mass distribution exhibits a large enhancement near threshold. The η -p and p-d mass distributions have been found to follow the phase space.
- The angular distributions of all three particles have been found to be nearly isotropic. However, the angular distribution of the η in the $p\ d \rightarrow {}^3\text{He}\ \eta$ reaction at a similar beam energy was found to be strongly forward peaked.
- A theoretical study of this reaction has been made by U. Tengblad, G. Gälldt and C. Wilkin. (EPJA 25 (2005) 267). In this work, the FSI have not been included.

A study of the $p\,d \rightarrow p\,d\,\eta$ reaction

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- We studied the $p\,d \rightarrow p\,d\,\eta$ reaction including the final state interactions.
- The η -d FSI has been incorporated through an η -d T-matrix which has been described in two ways:
 1. Factorized form \rightarrow An off-shell form factor \times on-shell part written in terms of effective range expansion up to fourth power in momentum.
The parameters of this expansion \rightarrow relativistic Faddeev equation solution for the ηNN system corresponding to different ηN scattering lengths.
 2. Solving few body equations in the finite rank approximation.
- The p-d FSI has been included (for both doublet and quadruplet state) by multiplying the production T-matrix by a inverse Jost Function (Watson-Migdal approach).



$$\langle |T_{pd \rightarrow pd\eta}| \rangle = \frac{3}{2} i \sum_{m's} \int \frac{d\vec{p}}{(2\pi)^3} \langle p n | d \rangle \langle |T_{pp \rightarrow \pi^+ d}| \rangle \frac{1}{k_\pi^2 - m_\pi^2 + i\epsilon} \langle |T_{\pi^+ n \rightarrow \eta p}| \rangle$$

Final state interaction

$$T_{\eta-d}(k, E(k_0), k') = g(k, k_0) T_{\eta-d}(E(k_0)) g(k', k_0)$$

on-shell $\eta - d$ T-matrix $F(k) = \left[\frac{1}{A} + \frac{1}{2} R k^2 + S k^4 - ik \right]^{-1}$

$$T_{\eta d}(k, k') = -\frac{1}{(2\pi)^2 \mu_{\eta d}} F_{\eta d}(k, E(k), k')$$

The half off-shell extrapolation factor $g(k', k_0)$

3. Factorized form for the $T_{\eta-d}$.

(i) off-shell form factor in terms of the deuteron form factor

$$g(k', k_0) = \int d\vec{r} j_0(rk'/2) \phi_d^2(r) j_0(rk_0/2)$$

(ii) ratio of the off-shell $\eta - d$ T-matrix to its on-shell value
both calculated using the three body equations within FRA

2. Few body equations within the finite rank approximation

- FRA \rightarrow restricting the spectral decomposition of the nuclear Hamiltonian in the intermediate state to the ground state, thereby neglecting all excited and break up channels of the nucleus
- deuteron break-up energy is just 2.225 MeV \rightarrow Applicability of the FRA may be limited

- Note: A comparative study of the η -d scattering length calculated in the FRA approximation and by solving exact AGS equations was made.
- The results were found to be very similar for the real part of the η -N scattering length < 0.5 fm.

The p-d final state interaction.

- We multiply the T-matrix by inverse Jost function which is written in terms of phase-shifts:

$$[J_o(k_{pd})]^{-2} = [J_o(k_{pd})]_Q^{-2} + [(1 + \frac{|E_B|}{E})J_o(k_{pd})]_D^{-2}$$

↑
effect of existence of a spin half bound state

- $E_B \longrightarrow$ Separation energy of ${}^3\text{He}$ in p-d

$$[J_o(k_{pd})]_Q^{-2} = \frac{(k_{pd}^2 + \alpha^2)^2 (b_Q^c)^2}{4} \times \frac{1}{3 C_o^2 k_{pd}^2} \left[\frac{2}{1 + \cot^2 \delta_Q} \right]$$

$$[J_o(k_{pd})]_D^{-2} = \frac{(k_{pd}^2 + \alpha^2)^2 (b_D^c)^2}{4} \times \frac{1}{3 C_o^2 k_{pd}^2} \left[\frac{1}{1 + \cot^2 \delta_D} \right]$$

$$\alpha = \left(\frac{1}{b_\mu^c} \right) \left[1 + \left(1 + \frac{2 b_\mu^c}{a_\mu^c} \right)^{\frac{1}{2}} \right]$$

$$\frac{1}{a_\mu^c} = \frac{1}{C_o^2} \left[\frac{1}{a_\mu} - 2 \gamma k_{pd} H_\gamma \right] \quad \mu = Q, D$$

Coulomb interaction

$$b_\mu^c = \frac{b_\mu}{C_o^2}$$

The phase shifts $\delta_{Q,D}$ are obtained from an effective-range expansion

$$C_o^2 k_{pd} \cot \delta_\mu = -\frac{1}{a_\mu} + \frac{1}{2} b_\mu k_{pd}^2 - 2 \gamma k_{pd} H_\gamma$$

Em coupling const

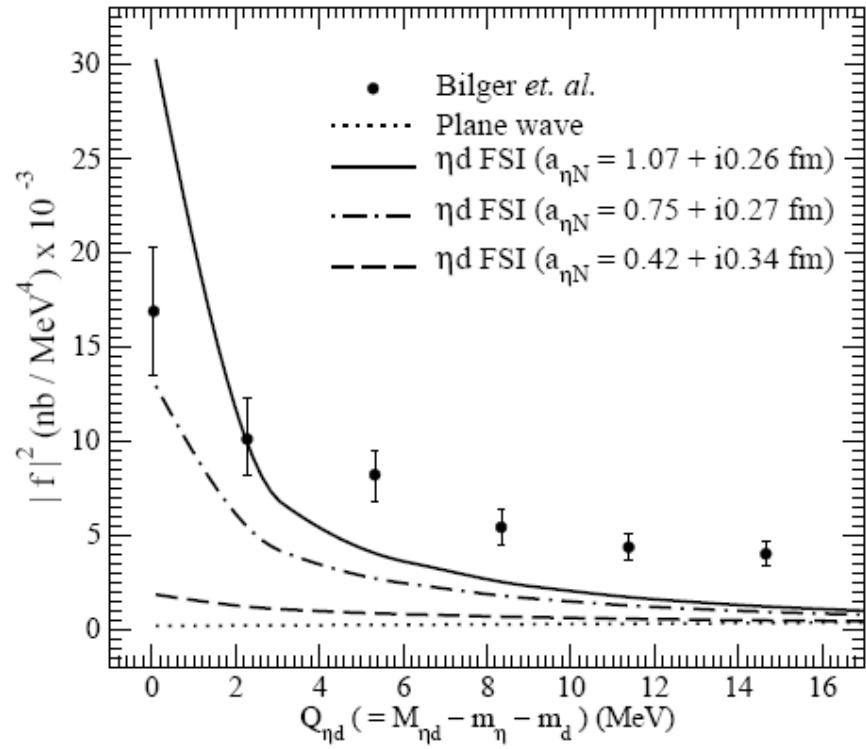
p-d reduced mass

Coulomb parameter

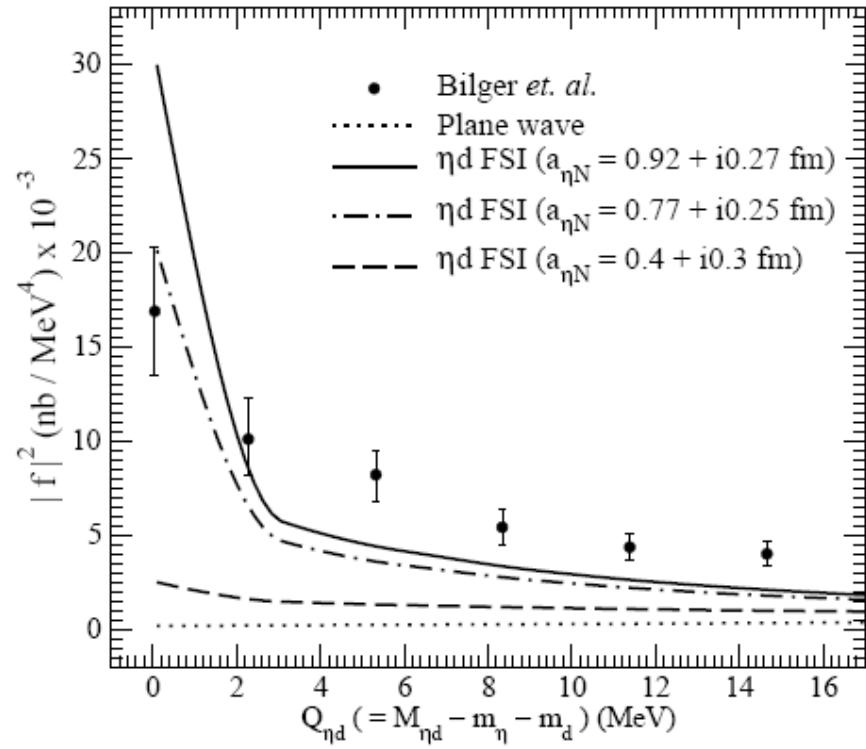
$$\gamma = \frac{\alpha m_{red}}{\hbar k_{pd}} \quad C_o^2 = \frac{2 \pi \gamma}{e^{2 \pi \gamma} - 1} \quad H_\gamma = \sum_{n=1}^{\infty} \frac{\gamma^2}{n(n^2 + \gamma^2)} - \ln(\gamma) - 0.57722$$

$a_Q = 11.88$ fm, $b_Q = 2.63$ fm, $a_D = 2.73$ fm, and $b_D = 2.27$ fm

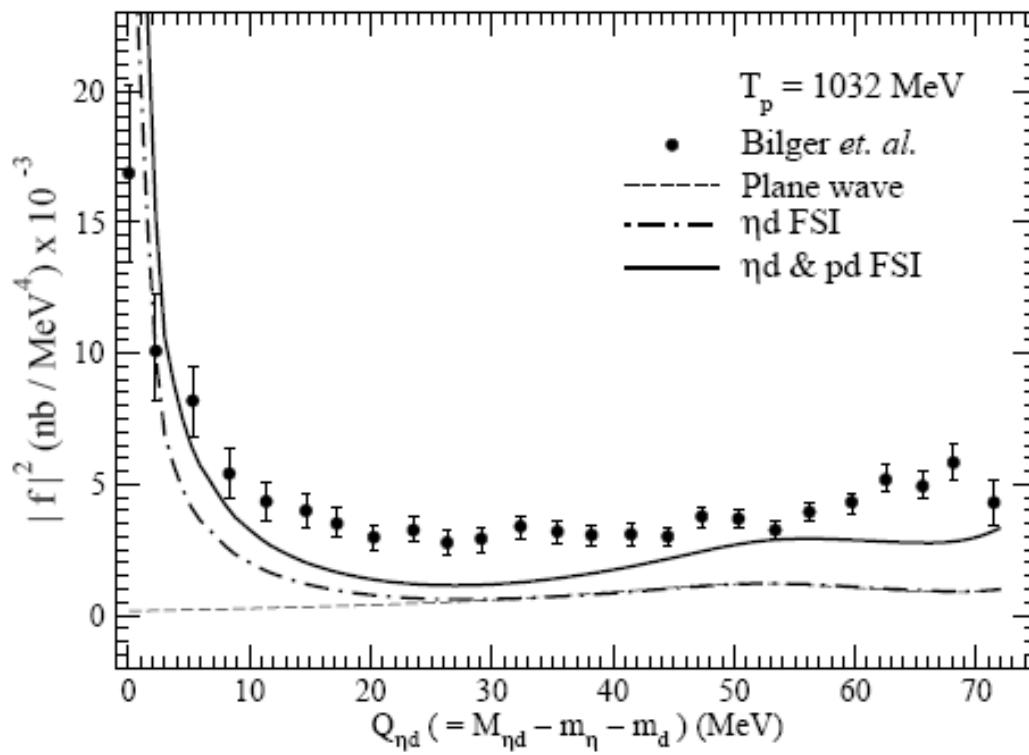
determined from a fit to the $p - d$ elastic-scattering phase shifts
in the relative $p - d$ momentum range up to around 200 MeV/c

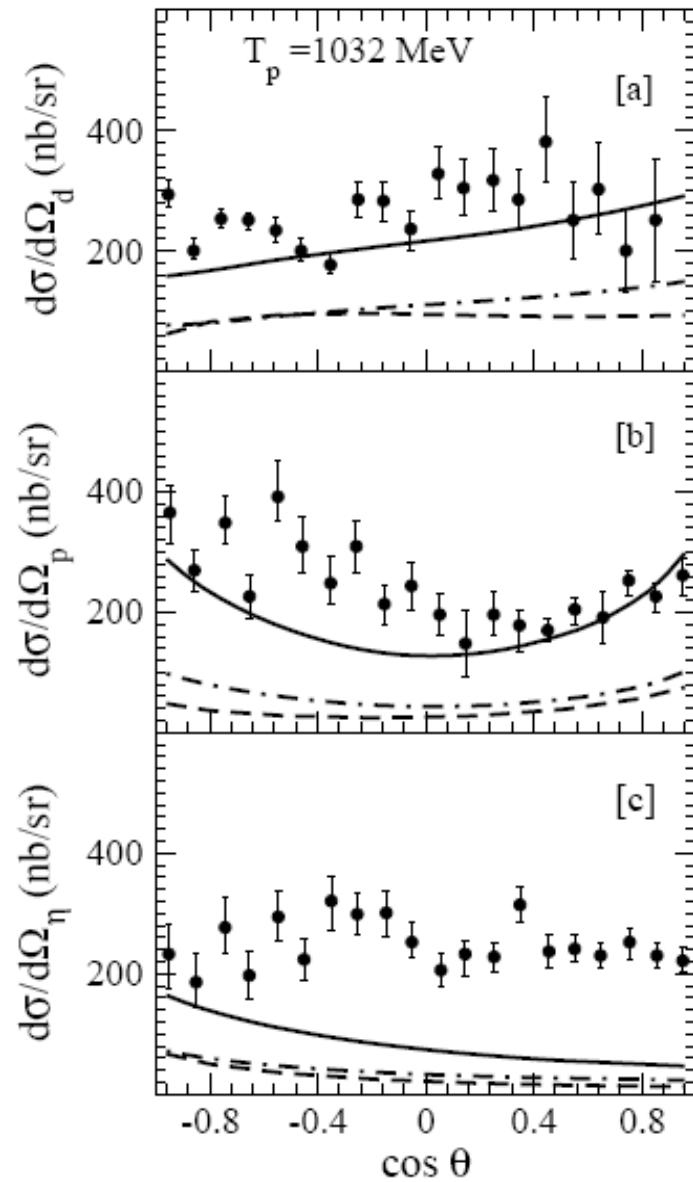


(a) Calculated results correspond to the factorized form of $T_{\eta d}$ with the off-shell factor generated from the deuteron form factor.



(b) Calculated results correspond to $T_{\eta d}$ obtained from few body equations within the FRA.





The ω and the η meson

	ω-meson	η-meson
■ Mass =	782.65 MeV	547.45 MeV
■ Isospin	0	0
■ G-parity	–	+
■ Spin	1	0
■ Parity	–	–
■ Charge conjugation	–	+
■ Decay modes:	$\pi^+ \pi^- \pi^0, \pi^0 \gamma$	$3\pi^0, 2\gamma$
■ Dynamics close to the threshold:	An interplay of multiple resonances expected.	Dominated by $S_{11}(1535)$
■ Scattering length: (fm)	$\text{Im}\{a_{\omega N}\} = 0.24^\dagger$ to 0.3 $\text{Re}\{a_{\omega N}\} = (-0.026)^\#$ to $(-0.44)^*$ to $(+1.6)^\S$	$\text{Im}\{a_{\omega N}\} \sim 0.24$ to 0.3^{++} $\text{Re}\{a_{\omega N}\} \sim 0.28$ to 1.0

[†]Hanhart et al. hep-ph/0107245,

[§]Klingl, Waas and Weise NPA 650 (1999) 299,

^{*}Lutz, Wolf and Friman NPA 706 (2002) 431,

[#] Shyklar, Lenske, Mosel and Penner PRC 71 (2005)

⁺⁺Sibirtsev et al. EPJA 22 (2004)495

- The experimental data close to threshold shows a large increase of the amplitude close to the threshold for the $p+d \rightarrow {}^3\text{He}+\eta$, $p+d \rightarrow p+d+\eta$ etc. reactions.
- In case of the $p+d \rightarrow {}^3\text{He}+\omega$ reaction, the amplitude shows a large dip close to the threshold.
- η - ${}^3\text{He}$ scattering length found to change sign. No information on the ω - ${}^3\text{He}$ scattering length.

New measurements for the p+d → ${}^3\text{He}+\omega$ reaction by the Celsius-Wasa Collaboration

- The angular distributions for the p+d → ${}^3\text{He}+\omega$ reaction have been measured by the Celsius-Wasa Collaboration (Karin Schönning et. al.) at two different beam energies, viz., 1360 MeV and 1450 MeV.
- $T_p = 1360 \text{ MeV}$ $\rightarrow k_\omega = 144 \text{ MeV/c}$ } momentum transfer
 $\sim 1110 \text{ MeV}$
 ~ 17 MeV above threshold
- $T_p = 1450 \text{ MeV}$ $\rightarrow k_\omega = 280 \text{ MeV/c}$ }
 $\sim 935 \text{ MeV}$
 ~ 155 MeV above threshold

We study this reaction in the two step model.

Coupled-channel analysis of ω -meson production in πN and γN reactions for c.m. energies up to 2 GeV

V. Shklyar,* H. Lenske, U. Mosel, and G. Penner

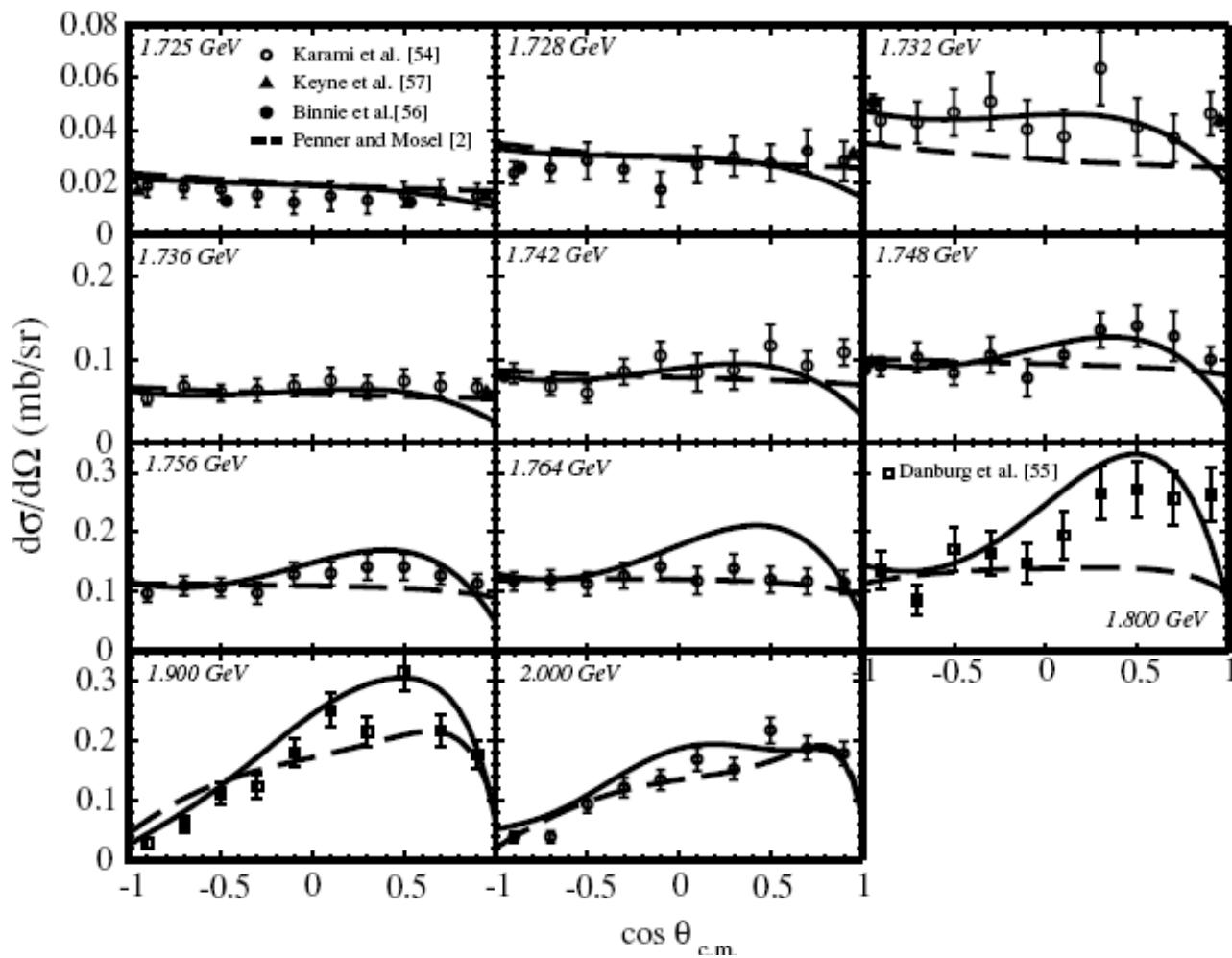
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(Received 3 December 2004; published 31 May 2005)

- A coupled-channel effective Lagrangian model
- Inclusion of γN , πN , $2\pi N$, ηN , ωN , $K\Lambda$, $K\Sigma$ final states
- Following 11 isospin $1/2$ resonances have been taken into account for the calculations:
 $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{11}(1650)$, $D_{15}(1675)$, $F_{15}(1680)$, $P_{11}(1710)$, $P_{13}(1720)$,
 $P_{13}(1900)$, $F_{15}(2000)$ and $D_{13}(2080)$.

Thus contributions from all important resonances up to 2 GeV has been included.

- A simultaneous analysis of the all the available data from threshold to 2 GeV has been made



Differential cross sections of the $\pi N \rightarrow \omega N$ reaction.

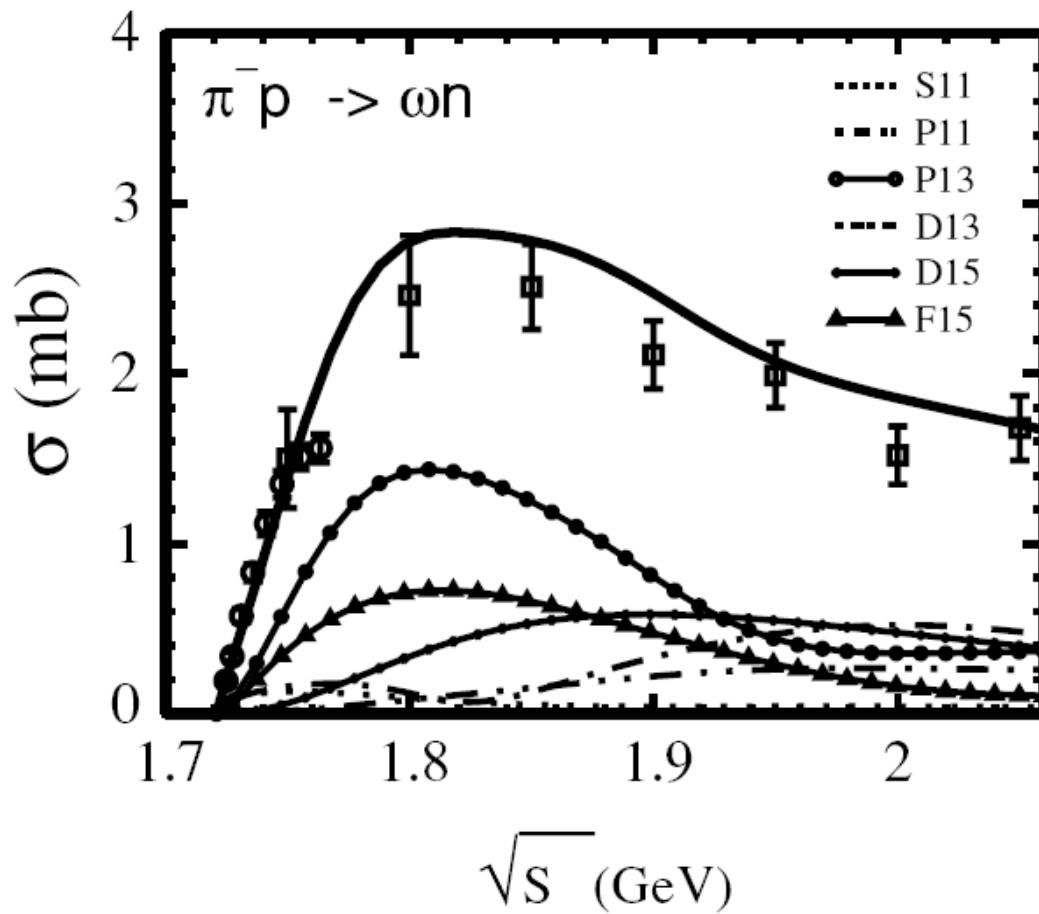
Exptl data from:

D. M. Binnie *et al.*, Phys. Rev. D **8**, 2789 (1973).

J. S. Danburg *et al.*, Phys. Rev. D **2**, 2564 (1970).

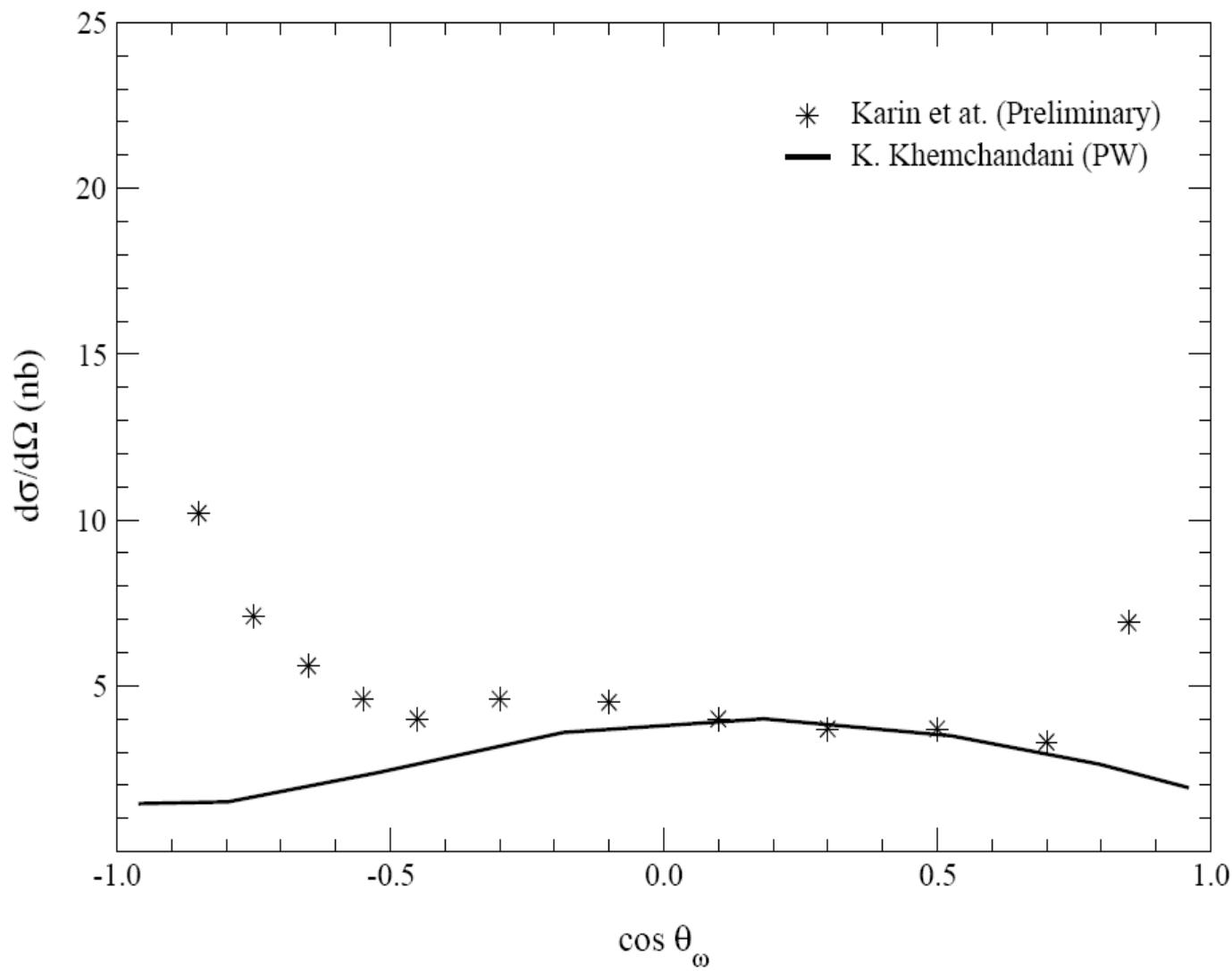
H. Karami *et al.*, Nucl. Phys. **B154**, 503 (1979).

J. Keyne *et al.*, Phys. Rev. D **14**, 28 (1976).

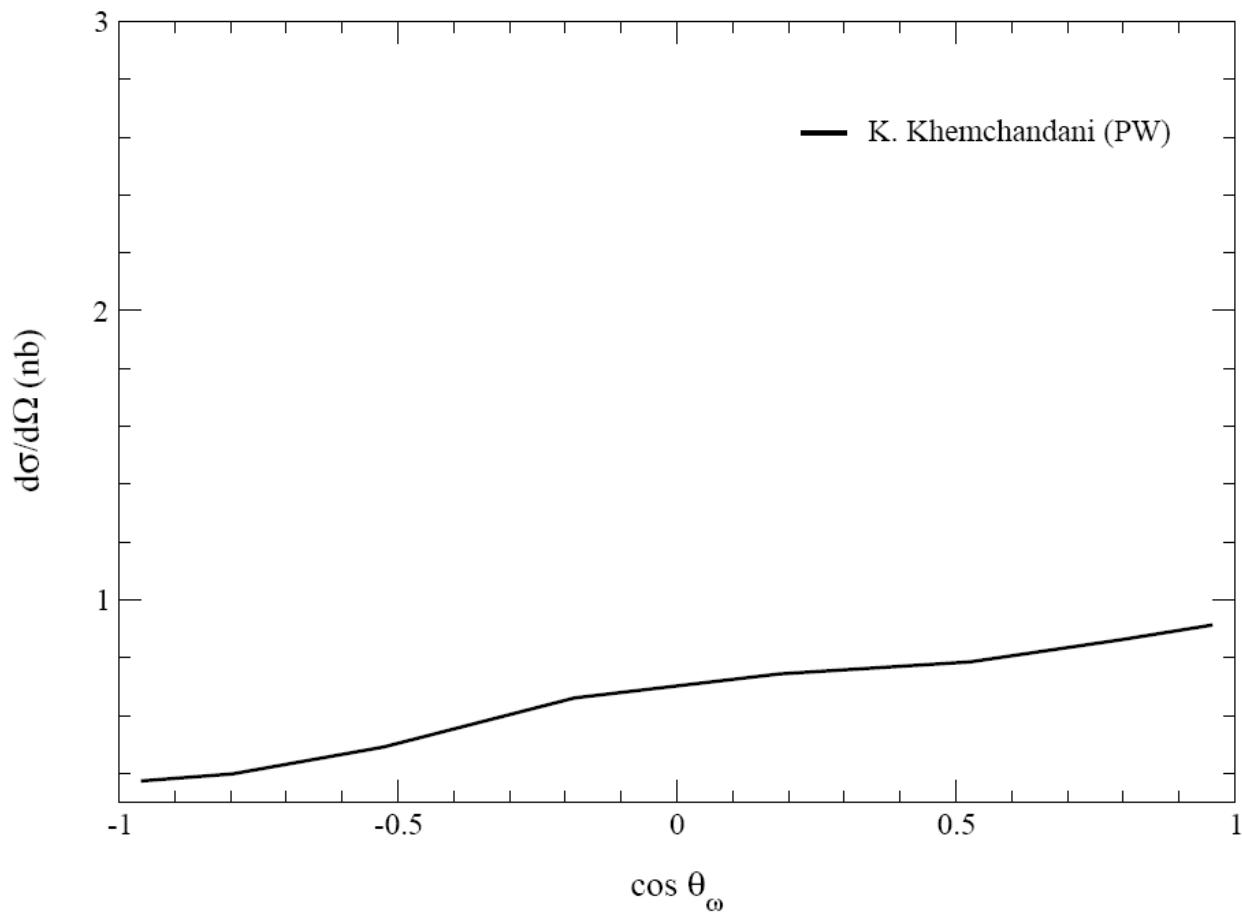


The partial wave decomposition of the total $\pi N \rightarrow \omega N$ cross-section.

$T_p = 1450$ MeV



$T_p = 1360 \text{ MeV}$



- I thank my collaborators (on the eta meson related studies):

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B.K.Jain (Mumbai Univ)

N.G.Kelkar (Universidad de los Andes, Bogotá)

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U.Mosel, H.Lenske

C.Hanhart