

**Mestrado Integrado em Engenharia Química**  
**Ano lectivo de 2012/2013, 1º semestre**

**Exame Normal de Física I**

11 de Janeiro de 2013

1. Numa das muitas experiências que foram mostradas nas aulas teóricas desta disciplina o Prof. Walter Lewin aqueceu água dentro duma lata metálica. Posteriormente, quando o vapor de água começou a sair da lata, ele fechou-a herméticamente com a água ainda a ferver e deixou-a uns bons minutos a arrefecer.
  - a Descreva pormenorizadamente, utilizando um diagrama de fase, o que sucedeu durante o processo de arrefecimento. Qual o objectivo desta experiência?  
Numa outra experiência um nadador mostrou a diferença de flexibilidade de uma bola plástica a profundidades diferentes dentro da água numa piscina.
  - b Qual o objectivo desta segunda experiência?  
Numa terceira experiência um assistente mostrou o que se passava com uma pequena quantidade de gelo seco (dióxido de carbono) dentro dum tubo plástico transparente e herméticamente fechado.
  - c Descreva pormenorizadamente, utilizando um diagrama de fase, o que sucedeu durante o processo de aquecimento. Qual o objectivo desta última experiência?
  
2.
  - a Uma escada de 5.0 metros com peso desprezável está encostada a uma parede vertical, sem atrito. A escada faz um ângulo de  $60^\circ$  com o chão horizontal. O coeficiente de atrito entre a escada e o chão é igual a 0.20.  
Até que altura pode, uma pessoa de 80 kg, subir a escada sem que ela deslize?
  - b Uma partícula pontual de massa 5.0 kg, que está ligada a um fio, realiza um movimento circular com 2.0 metros de raio e uma velocidade de 1.0 m/s sobre uma mesa sem atrito. O fio passa por um furo para baixo da mesa onde uma pessoa o segura. Descreva o que se passa com o movimento da partícula, deduzindo e analisando as fórmulas adequadas, quando a pessoa reduz o raio do movimento circular para 50 cm.
  
3.
  - a Num tubo fechado com 1.36 metros de comprimento onde se produz um som monocromático (som de uma só frequência) de 500 Hz são observados três nós. Determine a velocidade do som deduzindo e analisando as fórmulas necessárias.
  - b Considere duas fontes (altifalantes) de som monocromático com frequências ligeiramente diferentes e intensidades iguais. Os altifalantes estão situados um ao lado do outro. Descreva o ritmo do som ouvido por um observador que se situa a uma distância tão grande que se pode considerar nula a distância entre os altifalantes, deduzindo e analisando as fórmulas necessárias.

4. Considere uma gota de chuva esférica (diâmetro 5.0 mm) que cai verticalmente do céu para a terra. A força de resistência do ar é dada por

$$F_{\text{res}} = \frac{\pi}{2} C_d \rho_{\text{ar}} r^2 v^2$$

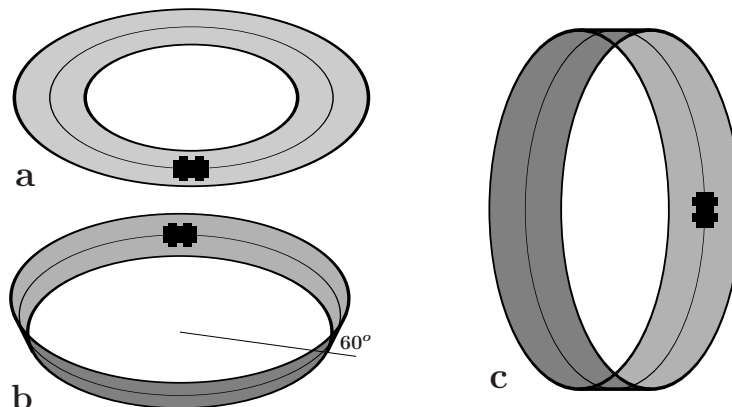
sendo  $C_d = \frac{24}{\text{Re}}$  no regime I e  $C_d = 0.47$  no regime II. A constante de Reynolds (Re) é dada por

$$\text{Re} = \frac{2vr\rho}{\mu}$$

onde  $v$  e  $r$  representam, respectivamente, a velocidade e o raio da gota de chuva e  $\rho$  e  $\mu$  representam, respectivamente, a densidade e o coeficiente de viscosidade do meio viscoso ( $\rho_{\text{ar}} = 1.275 \text{ kg/m}^3$  e  $\mu_{\text{ar}} = 1.983 \times 10^{-5} \text{ Pa s}$ ). Considere ainda que para valores do número de Reynolds abaixo de 500 se aplica o regime I e acima de 500 o regime II. Dentro dum carro que se move na horizontal parece que as gotas fazem ângulos de  $73^\circ$  com a vertical. Determine a velocidade do carro, deduzindo e explicando detalhadamente as fórmulas e o significado dos símbolos nelas usados.

5. Considere um carro que se desloca com uma velocidade constante ao longo dum percurso circular ...
- com raio de 30 metros, numa pista plana e horizontal. Qual a velocidade máxima para que o carro não saia do percurso, se o coeficiente de atrito entre os pneus e a pista for igual a 0.4?
  - e horizontal com raio de 30 metros, numa pista cónica que faz um ângulo de  $60^\circ$  com a horizontal. Qual a velocidade máxima para que o carro não saia do percurso, se o coeficiente de atrito entre os pneus e a pista for igual a 0.4?
  - e vertical com raio de 5.0 metros, dentro de um cilindro deitado. Qual a velocidade mínima necessária para que o carro não perca o contacto com o cilindro no topo do percurso?

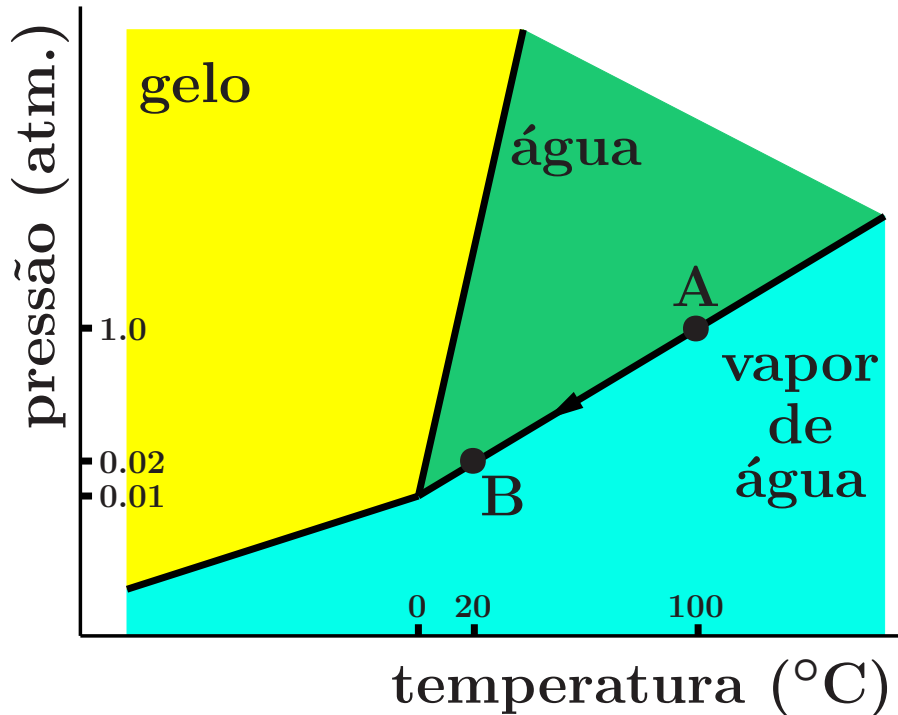
Considere o carro pontual nas três alíneas. Deduza e explique em pormenor as fórmulas e o significado dos símbolos nelas usados.



# Solutions

## Exercício 1

a:



As the above phase diagram shows, at a pressure of 1 atm. boiling water has a temperature of 100 °C. This situation is indicated by *A* in the above phase diagram.

Walter Lewin closes the metal container when the water vapor has substituted almost all air in the container. In contact with the surrounding air at room temperature (20 °C), the system will slowly cool down to room temperature. But, all the time the container contains water and water vapor in coexistence. Hence, the process follows the curve which indicates the temperature and the pressure for the situation that water and water vapor are in coexistence. This is indicated by an arrow in the above phase diagram. The final temperature will be room temperature.

At room temperature, indicated by *B* in the above phase diagram, a mixture of water and water vapor has a pressure of about 0.02 atm. That is much less than the pressure of the environment which is about 1.0 atm. Hence, the pressure of the interior of the container is much less than the pressure of the exterior of the container, which causes the container to implode as the video indeed shows.

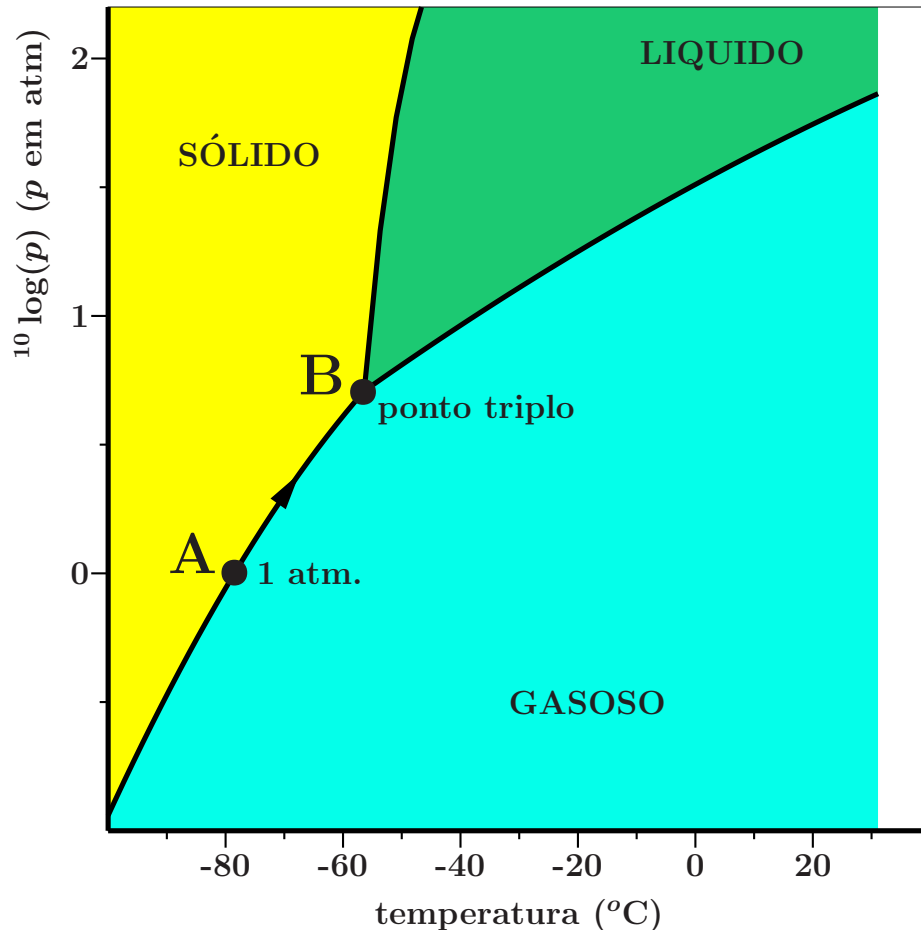
The purposes of this experiment are:

1. To show that the boiling point of water at 20 °C has a very low pressure of about 0.02 atm.
2. To demonstrate the tremendous force of the atmospheric pressure.

**b:** The plastic ball consists of a thin plastic layer which encloses a certain amount of air that has the same pressure, or a little bit higher, as the environmental pressure of 1 atm. That is the reason that one needs quite some force in order to deform the plastic ball. But, at the bottom of the swimming pool the ball suffers an exterior pressure which is a lot larger than the environmental pressure of 1 atm., namely about 0.1 atm extra for each 1 meter of depth under water. Under such pressure the air in the interior of the ball occupies a smaller volume. Hence, the plastic layer

is too large for the volume of air it encloses. As a consequence it can be more easily deformed. The purpose of the experiment is to demonstrate that the volume of a certyain amount of (ideal) gas decreases when the pressure of the amount of gas increases. For an ideal gas this is given by Boyle's law.

c:



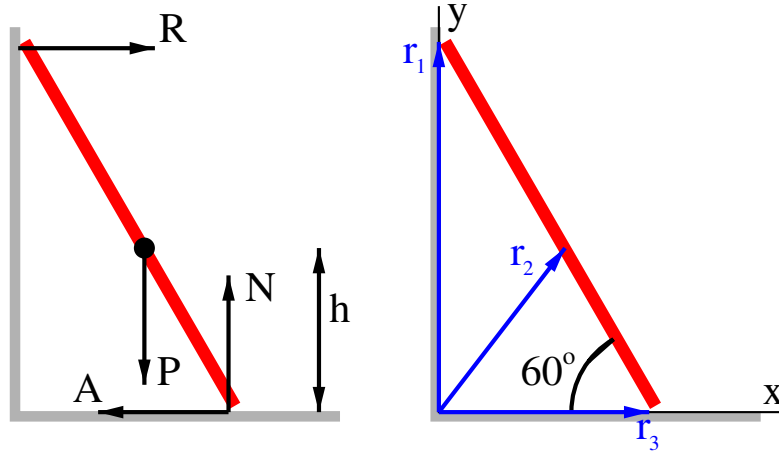
As the above phase diagram shows, at a pressure of 1 atm. solid CO<sub>2</sub> can at most have a temperature of some  $-78.5\text{ }^{\circ}\text{C}$ . At that temperature solid CO<sub>2</sub> sublimates to gaseous CO<sub>2</sub> when in thermal contact with substances of higher temperatures, like with the air or with a plastic tube at room temperature. Consequently, inside the plastic tube, but before closing the tube, one has an equilibrium of solid and gaseous CO<sub>2</sub> at a temperature of  $-78.5\text{ }^{\circ}\text{C}$  and at a pressure of 1.0 atm. This situation is indicated by A in the above phase diagram. After closing the tube, the solid CO<sub>2</sub> recieves heat from its surroundings, hence its temperature increases. But all the time it is sublimating into gaseous CO<sub>2</sub> when it recieves heat. So, the process is indicated by the curve at which solid and gaseous CO<sub>2</sub> are in coexistence. This is indicated by an arrow in the above phase diagram.

We find that as the temperature increases also the pressure increases.

At a certain instant one reaches, at a temperature of  $-56.4\text{ }^{\circ}\text{C}$  and a pressure of 5.4 atmosphere, the triple point of CO<sub>2</sub>, which is the purpose of the experiment and which is indicated by B in the above phase diagram. At that point also liquid CO<sub>2</sub> can exist and is indeed observed inside the plastic tube.

## Exercício 2

a:



In the figure we show the four forces which are involved in this case: The weight of the person  $\vec{P}$ , the normal reaction force of the floor  $\vec{N}$ , the friction force between the ladder and the floor  $\vec{A}$  and the normal reaction force of the wall  $\vec{R}$ . In the case of point particles all forces apply in the same point. However, in the case of extended objects the forces have different points of application. In the righthand figure we have indicated the positions where the four forces apply:  $\vec{r}_1$  where the normal reaction force of the wall  $\vec{R}$  applies,  $\vec{r}_2$  where the weight of the person  $\vec{P}$  applies and  $\vec{r}_3$  where the normal reaction force of the floor  $\vec{N}$  and the friction force between the ladder and the floor  $\vec{A}$  apply.

We have chosen the  $xy$  coordinates as indicated in the righthand figure. The resulting vector representations for the four forces are then given by

$$\vec{P} = \begin{pmatrix} 0 \\ -mg \end{pmatrix}, \quad \vec{N} = \begin{pmatrix} 0 \\ N \end{pmatrix}, \quad \vec{R} = \begin{pmatrix} R \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{A} = \begin{pmatrix} -A \\ 0 \end{pmatrix}$$

Let us denote the height where the person stays on the ladder by  $h$  and, furthermore, the length of the ladder by  $\ell$  ( $\ell = 5$  m). Then, for the distance from the origin of our coordinate system to the place where the ladder touches the floor, one finds  $\ell \cos(60^\circ) = \ell/2$ , whereas the ladder touches the wall at a height of  $\ell \sin(60^\circ) = \ell\sqrt{3}/2$ . Consequently,

$$\vec{r}_1 = \begin{pmatrix} 0 \\ \frac{1}{2}\ell\sqrt{3} \end{pmatrix}, \quad \vec{r}_2 = \begin{pmatrix} \frac{1}{2}\ell - \frac{h}{\sqrt{3}} \\ h \end{pmatrix} \quad \text{and} \quad \vec{r}_3 = \begin{pmatrix} \frac{1}{2}\ell \\ 0 \end{pmatrix}$$

When there is equilibrium, then the forces cancel each other (first law of Newton). Hence:

$$0 = \vec{P} + \vec{N} + \vec{R} + \vec{A} = \begin{pmatrix} R - A \\ -mg + N \end{pmatrix}$$

From which we obtain

$$A = R \quad \text{and} \quad N = P = mg$$

Consequently, the forces  $\vec{R}$  and  $\vec{A}$  are undetermined.

But, unlike point particles, extended objects can rotate. They do so when a torque is acting on them. In equilibrium, when nothing rotates, there is no torque acting on the ladder. Hence,

for the torques of the four forces one has

$$\begin{aligned}
 0 &= \vec{r}_1 \times \vec{R} + \vec{r}_2 \times \vec{P} + \vec{r}_3 \times \vec{N} + \vec{r}_3 \times \vec{A} \\
 &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\ell\sqrt{3}R \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\left(\frac{1}{2}\ell - \frac{h}{\sqrt{3}}\right)mg \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2}\ell N \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\ell\sqrt{3}A - \left(\frac{1}{2}\ell - \frac{h}{\sqrt{3}}\right)N + \frac{1}{2}\ell N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\ell\sqrt{3}A + \frac{h}{\sqrt{3}}N \end{pmatrix}
 \end{aligned}$$

Notice that the torque vectors associated with anticlockwise rotation are in the positive  $z$  direction, whereas those associated with clockwise rotation are in the negative  $z$  direction.

We find thus:

$$\frac{A}{N} = \frac{2h}{3\ell}$$

Furthermore we know that

$$\frac{2h}{3\ell} = \frac{A}{N} \leq (\text{static friction coefficient})$$

Consequently, the ladder starts slipping away when

$$\frac{2h}{3\ell} = (\text{static friction coefficient}) = 0.20 \iff h = 0.20 \frac{3}{2}\ell = 0.30 \times (5.0 \text{ m}) = 1.5 \text{ m}.$$

**b:** The force with which the person pulls the cord is transmitted to the point particle by the tension in the cord. However, the position vector of the point particle and the tension in the cord point in the same direction. That implies that the vectorial product of the position vector of the point particle and the tension in the cord vanishes. Hence, no torque is acting on the system. Consequently, the angular momentum of the system is conserved.

The angular momentum of the rotating point particle is given by ( $I$  moment of inertia of the system,  $\omega$  angular velocity of the point particle,  $M$  mass of the point particle  $R$  radius of the trajectory of the point particle and  $v$  velocity of the point particle.)

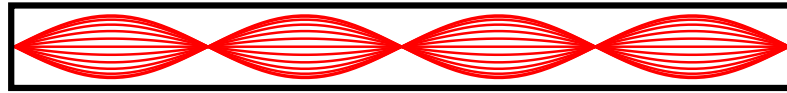
$$L = I\omega = MR^2 \frac{v}{R} = MRv$$

If that is conserved, one has ( $M$  is constant)

$$R_{\text{final}}v_{\text{final}} = R_{\text{inicial}}v_{\text{inicial}} \iff v_{\text{final}} = \frac{R_{\text{inicial}}}{R_{\text{final}}} v_{\text{inicial}} = \frac{2.0 \text{ m}}{0.5 \text{ m}} (1.0 \text{ m/s}) = 4.0 \text{ m/s}.$$

### Exercício 3

**a:** For standing waves in a closed tube one assumes that at the ends of the closed tube are vibration nodes. Consequently, when there are three additional nodes observed then the tube has five nodes. That situation is depicted in the figure below.



The distance between two nodes is equal to half a wave length. Hence, the length of the tube  $\ell$  equals two wave lengths:

$$\ell = 2\lambda \iff \lambda = \frac{1}{2}\ell = \frac{1}{2}(1.36 \text{ m}) = 0.68 \text{ m} \quad .$$

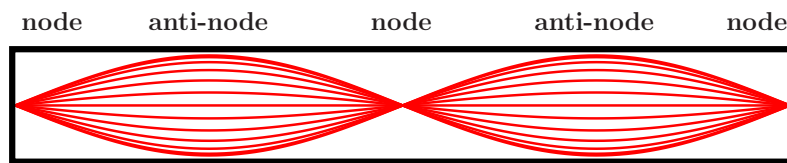
The speed of sound  $v$  can be found from the distance sound travels during one oscillation, namely exactly one wave length  $\lambda$ . The time lapse for one oscillation equals the period  $T$ . Consequently, we obtain

$$\lambda = vT \iff v = \frac{\lambda}{T} \quad .$$

The frequency  $f$  defines the number of oscillations per second, hence  $fT = 1$ .

$$v = \frac{\lambda}{T} = f\lambda = (500 \text{ Hz})(0.68 \text{ m}) = 340 \text{ m/s} \quad .$$

Some students assumed that the three nodes included the nodes at the closed ends of the tube. That situation is show in the figure below



In that case the length of the tube equals one wave length and thus the speed of sound would give 680 m/s for that situation.

**b:** An observer hears a mixture of the sound from one loudspeaker (1) and of the sound from the other loudspeaker (2). Let the rithme from 1 be given by  $\omega_1$  and from 2 by  $\omega_2$ . So, an observer hears a signal which is composed of two sinusoidal with different frequencies, but with the same amplitudes  $A$ :

$$A \sin(\omega_1 t) + A \sin(\omega_2 t) = 2A \cos\left(\frac{1}{2}(\omega_1 - \omega_2)t\right) \sin\left(\frac{1}{2}(\omega_1 + \omega_2)t\right) \quad .$$

Now, for a small difference between  $\omega_1$  and  $\omega_2$  one has

$$\frac{1}{2}(\omega_1 + \omega_2) \approx \omega_1 \quad .$$

Hence, the rithme of the sine part of the signal is about the same as the sound which comes from either of the two loudspeakers, which is a rithme which is too fast to be heard as individual oscillations. One just hears a certain sound of a certain frequency.

But, for a small difference between  $\omega_1$  and  $\omega_2$  one also has that  $\omega_1 - \omega_2$  is small. That implies that the cosine part varies very slowly with time. Such oscillation can be slow enough to be observed by our hearing. That effect is called "beats": <http://www.youtube.com/watch?v=gaFF4gaiKJ8>

## Exercício 4

The terminal velocity is the maximum velocity of a rain droplet. Hence, when the terminal velocity is reached the velocity of a droplet is constant, consequently the acceleration zero and thus one has the following balance of forces. Downward acts the weight of the droplet, whereas upward act the air resistance and the buoyant force (Arquimedes):

$$\text{Weight} = F_{\text{res}} + F_{\text{Arquimedes}}$$

The weight of the water droplet equals its mass times the gravitational constant  $g$ , whereas its mass equals its volume  $V$  times the density of water  $\rho_{\text{água}}$ . Furthermore, the force of the air resistance is given in terms of the drag constant  $C_d$ , the density of air  $\rho_{\text{ar}}$ , the radius of the spherical droplet  $r$  and its velocity  $v = v_{\text{term}}$ . Finally, the buoyant force  $F_{\text{Arquimedes}}$  equals the weight of the displaced air, which, in its turn, equals the volume  $V$  of the droplet times the density of air  $\rho_{\text{ar}}$  times the gravitational constant  $g$ .

$$V\rho_{\text{água}}g = \frac{1}{2}\pi C_d\rho_{\text{ar}}r^2v_{\text{term}}^2 + V\rho_{\text{ar}}g$$

We obtain the terminal velocity by rearranging the various terms and by substitution of the volume  $V$  of the sphere by  $V = \frac{4}{3}\pi r^3$ .

$$\frac{4}{3}\pi r^3(\rho_{\text{água}} - \rho_{\text{ar}})g = \frac{1}{2}\pi C_d\rho_{\text{ar}}r^2v_{\text{term}}^2$$

$$8r(\rho_{\text{água}} - \rho_{\text{ar}})g = 3C_d\rho_{\text{ar}}v_{\text{term}}^2$$

From here on it depends on the expression for  $C_d$  on how to proceed.

In **regime I** one has the following relation between  $C_d$  and Reynolds constant,  $\text{Re}$ , whereas, moreover, the relation between  $\text{Re}$  and  $v_{\text{term}}$ ,  $r$ ,  $\rho_{\text{ar}}$  and  $\mu_{\text{ar}}$  is given.

$$C_d = \frac{24}{\text{Re}} = \frac{24\mu_{\text{ar}}}{2v_{\text{term}}r\rho_{\text{ar}}}$$

When we substitute this in the previous expression, we find

$$8r(\rho_{\text{água}} - \rho_{\text{ar}})g = 3\frac{24\mu_{\text{ar}}}{2v_{\text{term}}r\rho_{\text{ar}}}\rho_{\text{ar}}v_{\text{term}}^2 = \frac{36\mu_{\text{ar}}}{r}v_{\text{term}}^2$$

Here, we solve for  $v_{\text{term}}$

$$v_{\text{term}} = \frac{2}{9}r^2\frac{\rho_{\text{água}} - \rho_{\text{ar}}}{\mu_{\text{ar}}}g$$

Hence, we obtain for the Reynolds number in this case

$$\text{Re} = \frac{2v_{\text{term}}r\rho_{\text{ar}}}{\mu_{\text{ar}}} = \frac{4}{9}r^3\frac{\rho_{\text{água}} - \rho_{\text{ar}}}{\mu_{\text{ar}}^2}\rho_{\text{ar}}g$$

We evaluate the expression

$$\begin{aligned} \text{Re} &= \frac{4}{9}\left(2.5 \times 10^{-3} \text{ m}\right)^3 \frac{(1.0 \times 10^3 - 1.275) \text{ kg/m}^3}{(1.983 \times 10^{-5} \text{ Pa s})^2} (1.275 \text{ kg/m}^3) (9.83 \text{ m/s}^2) \\ &= 2.21 \times 10^5 \end{aligned}$$



We find thus that the Reynolds number for this case is much larger than what is allowed to apply regime I.

In **regime II** one has

$$C_d = 0.47$$

which leaves us for  $v_{\text{term}}$  with the expression

$$v_{\text{term}}^2 = \frac{\frac{4}{3}\pi r^3 (\rho_{\text{água}} - \rho_{\text{ar}}) g}{\frac{1}{2}\pi C_d \rho_{\text{ar}} r^2} = \frac{8r (\rho_{\text{água}} - \rho_{\text{ar}}) g}{3C_d \rho_{\text{ar}}} = \frac{8r (\rho_{\text{água}} - \rho_{\text{ar}}) g}{3C_d \rho_{\text{ar}}}$$

We evaluate

$$v_{\text{term}}^2 = \frac{8 (2.5 \times 10^{-3} \text{ m}) (1.0 \times 10^3 - 1.275) \text{ kg/m}^3 (9.83 \text{ m/s}^2)}{3 \times 0.47 \times (1.275 \text{ kg/m}^3)}$$

We find  $v_{\text{term}} = 10.45 \text{ m/s}$ .

Hence, for the Reynolds number

$$\begin{aligned} \text{Re} &= \frac{2v_{\text{term}} r \rho_{\text{ar}}}{\mu_{\text{ar}}} = \frac{2 (10.45 \text{ m/s}) (2.5 \times 10^{-3} \text{ m}) (1.275 \text{ kg/m}^3)}{(1.983 \times 10^{-5} \text{ Pa s})} \\ &= 3.35 \times 10^3 \end{aligned}$$

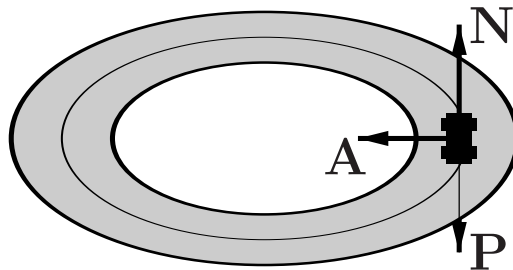
This is larger than 500. Hence, we are allowed to apply regime II for this case.

For the velocity of the car we obtain then finally

$$v_{\text{car}} = v_{\text{term}} \times \tan(73^\circ) = 10.45 \text{ m/s} \times \tan(73^\circ) = 34.2 \text{ m/s} = 123 \text{ km/h}$$

## Exercício 5

a:



As the above figure shows, the weight ( $m$  mass,  $g$  gravitational acceleration)  $P = mg$  of the point particle (car) is compensated by the normal force  $N = P$ . The force which keeps the point particle in circular motion is the friction force  $A$  between the tires and the track. Consequently, the centripetal force ( $R$  radius,  $v$  velocity)  $F_{\text{centripetal}} = mv^2/R$  equals the friction force:

$$m \frac{v^2}{R} = F_{\text{centripetal}} = A \quad .$$

The friction force is always smaller than or equal to the maximum friction force  $A_{\max}$ , which is here given by

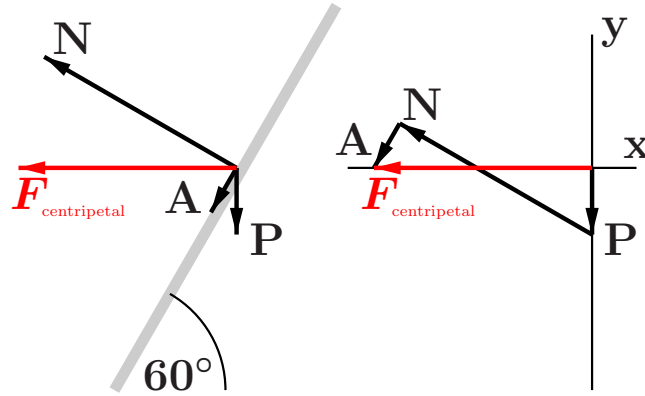
$$A_{\max} = (\text{friction coefficient}) \times N = 0.4N = 0.4P = 0.4mg \quad .$$

So, we obtain

$$m \frac{v_{\max}^2}{R} = A \leq A_{\max} = 0.4mg \quad \iff \quad v_{\max} \leq \sqrt{0.4gR} = \sqrt{0.4 (9.83 \text{ m/s}^2) (30.0 \text{ m})} = 10.9 \text{ m/s} \quad .$$

The maximum velocity for the car not to leave the circular motion equals  $v_{\max} = 10.9 \text{ m/s} = 39.1 \text{ km/h}$ .

b:



As the above figure shows, the weight ( $m$  mass,  $g$  gravitational acceleration)  $P = mg$  of the point particle (car) is not compensated by the normal force  $N$ . However, since the particle's trajectory is horizontal, the sum of  $P$ ,  $N$  and the friction force  $A$  must be horizontal and equal to the centripetal force  $F_{\text{centripetal}}$ . We write ( $N = |\vec{N}|$  and  $A = |\vec{A}|$ )

$$\vec{P} = \begin{pmatrix} 0 \\ -mg \end{pmatrix} \quad , \quad \vec{N} = \begin{pmatrix} -N \sin(60^\circ) \\ N \cos(60^\circ) \end{pmatrix} \quad , \quad \vec{A} = \begin{pmatrix} -A \cos(60^\circ) \\ -A \sin(60^\circ) \end{pmatrix}$$

and ( $R$  radius and  $v$  velocity)

$$F_{\text{centripetal}} = \begin{pmatrix} -m \frac{v^2}{R} \\ 0 \end{pmatrix} \quad .$$

So,

$$\begin{pmatrix} -m \frac{v^2}{R} \\ 0 \end{pmatrix} = F_{\text{centripetal}} = \vec{P} + \vec{N} + \vec{A} = \begin{pmatrix} -N \sin(60^\circ) - A \cos(60^\circ) \\ -mg + N \cos(60^\circ) - A \sin(60^\circ) \end{pmatrix} \quad .$$

From the  $y$  component we deduce

$$mg = N \cos(60^\circ) - A \sin(60^\circ) \quad .$$

From the  $x$  component we thus find

$$m \frac{v^2}{R} = N \sin(60^\circ) + A \cos(60^\circ) \quad \iff$$

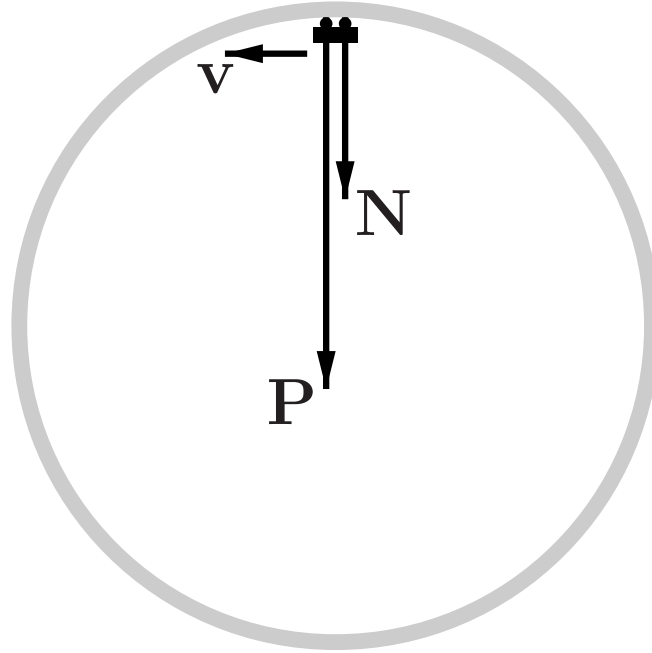
$$\iff v^2 = \frac{Rg}{mg} \{N \sin(60^\circ) + A \cos(60^\circ)\} = Rg \frac{N \sin(60^\circ) + A \cos(60^\circ)}{N \cos(60^\circ) - A \sin(60^\circ)} .$$

We obtain the maximum velocity when we substitute the maximum possible friction force  $A_{\max} = \alpha N$  ( $\alpha$  friction coefficient).

$$\iff v^2 = Rg \frac{\sin(60^\circ) + \alpha \cos(60^\circ)}{\cos(60^\circ) - \alpha \sin(60^\circ)} = (30.0 \text{ m}) (9.83 \text{ m/s}^2) \frac{\frac{1}{2}\sqrt{3} + 0.4 \times \frac{1}{2}}{\frac{1}{2} - 0.4 \times \frac{1}{2}\sqrt{3}} = (45.2 \text{ m/s})^2 .$$

The maximum velocity for the car not to leave the circular motion equals  $v_{\max} = 45.2 \text{ m/s} = 163 \text{ km/h}$ .

c:



As the above figure shows, when the point particle (car) passes through the top of its circular trajectory the weight ( $m$  mass,  $g$  gravitational acceleration)  $P = mg$  of the point particle (car) and the normal force  $N$  point in the same direction, both towards the center of the circle. Consequently, the centripetal force is equal to the sum of those two forces ( $v$  velocity,  $R = 5 \text{ m}$  radius).

$$m \frac{v^2}{R} = F_{\text{centripetal}} = P + N = mg + N .$$

The normal force indicates whether there is contact between the car and the wall of the cylinder. Hence,

$$N > 0 .$$

The minimum velocity at which contact is just lost in the top, occurs for  $N = 0$ . In that case

$$m \frac{v_{\min}^2}{R} = F_{\text{centripetal}} = P = mg \iff v_{\min} = \sqrt{gR} = \sqrt{(9.83 \text{ m/s}^2)(5.0 \text{ m})} = 7.0 \text{ m/s} .$$

Thus, the minimum velocity equals  $v_{\min} = 7.0 \text{ m/s} = 25 \text{ km/h}$ .