Mestrado Integrado em Engenharia Química
Ano lectivo de 2013/2014, 1^o semestre

Exame Normal de Física I
13 de Janeiro de 2014

Sempre apresenta claramente o seu raciocínio
e justifica as fórmulas e os símbolos nelas usados.

Para a constante da aceleração gravítica utilizar o valor de $g = 9.83$ m/s^2.

1. Considere um comboio de alta velocidade que emite um sinal sonoro de 55 Hz (A_1 na escala musical) e que se aproxima, com uma velocidade retilínio uniforme de 360 km/h, de uma passagem de nível onde se encontra uma pessoa (A) à espera junto das cancelas fechadas.

a. Determine a fracção Δx da distância $x - \Delta x$ da pessoa (A) onde o comboio se encontra quando emite o fim de uma oscilação completa do som, caso o comboio tenha emitido o início da oscilação completa em causa a uma distância x da pessoa (A).

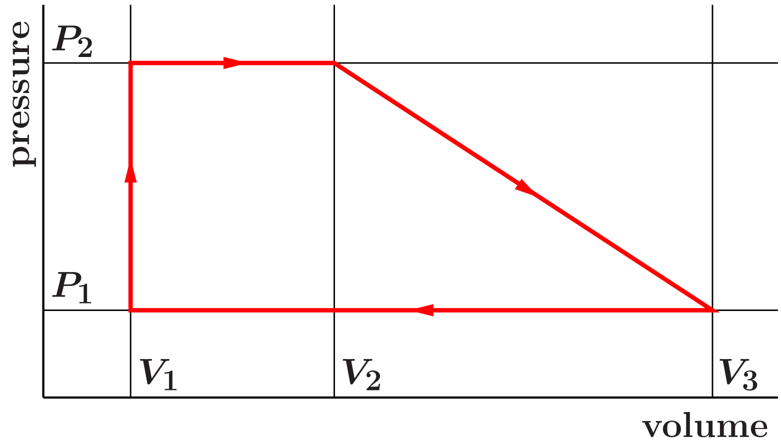
b. Determine a diferença de tempo entre as chegadas no ouvido da pessoa (A) do início e do fim da oscilação completa referida na alínea **a**. A velocidade do som no ar é igual a 340 m/s.

c. Qual a frequência do som ouvido pela pessoa (A)? O seu valor significa que a pessoa (A) recebe um som mais agudo, mais grave ou igual ao som emitido?

2. Uma escada de 5.0 metros, com peso desprezável, está encostada a uma parede vertical. A escada faz um ângulo de 60° com o chão horizontal. Os coeficentes de atrito entre a escada e o chão e entre a escada e a parede são respectivamente iguais a 0.20 e 0.50. Até que altura pode uma pessoa de 65 kg subir a escada sem que ela deslize?

3. Um cilindro maciço, homogéneo, com raio da base R e massa m rola por uma descida abaixo, sem deslizar. A descida faz um ângulo de 30° com a horizontal e o momento de inércia do cilindro é dado por $\frac{1}{2}mR^2$. Determine a aceleração do cilindro.

4. Considere um cilindro de um motor de combustão cujo funcionamento é representado pelo diagrama P-V da figura seguinte.



Considere também as seguintes relações: $V_2 = 1.5V_1$, $V_3 = 2.5V_1$ e $P_2 = 3P_1$.

a Se a temperatura no ponto (V_1,P_1) é igual a 300° K, qual é a temperatura nos pontos (V_1,P_2), (V_2,P_2) e (V_3,P_1)?

b Determine a quantidade de calor, em unidades P_1V_1, fornecida ao/pelo combustível nas transformações $(V_1, P_1) \to (V_1, P_2)$, $(V_1, P_2) \to (V_2, P_2)$, $(V_2, P_2) \to (V_3, P_1)$ e $(V_3, P_1) \to (V_1, P_1)$.

c Considere a eficácia de um motor definida como o quociente entre o trabalho efectuado e o calor fornecido. Determine a eficácia do motor representado pelo diagrama P-V em causa.

5. Considere três esferas ocas de metal de massas iguais (de 5.245 kg) mas com raios diferentes, respectivamente de, 8.42 cm, 10.69 cm e 12.24 cm, colocadas na superfície da água do mar (densidade 1.025 g/cm^3).

a Explique o que acontece com as esferas após serem libertadas.

b A esfera com raio inferior resiste a uma pressão máxima de 10^7 Pa. Qual é a profundidade do mar onde esta esfera implode?

c Considere a dinâmica do movimento da esfera com raio inferior dada por

$$M\frac{dv(t)}{dt} = -Mg + gV\rho_{\text{água do mar}} + C_2 r^2 v^2(t) \quad ,$$

onde $v(t)$ representa a velocidade da esfera, r o seu raio, $V = 4\pi r^3/3$ o seu volume e M a sua massa. Considere também a constante de viscosidade $C_2 = 730$ kg/m^3. Assumindo que a velocidade terminal é alcançada num espaço de tempo desprezável, determine o tempo que a esfera demora a alcançar a profundidade determinada na alínea **b**.

Solutions

Exercício 1

a: One complete oscillation is performed in a time $T = 1/f$, where f represents the frequency of the sound. In that time the train moves a distance given by

$$\Delta x = v_{\text{train}} T = \frac{v_{\text{train}}}{f} = \frac{360 \text{ km/h}}{55 \text{ Hz}} = \frac{100 \text{ m/s}}{55 \text{ s}^{-1}} = 1.82 \text{ m}$$

b: Let us define the time interval between the arrival of the onset of the complete oscillation and the arrival of the end of it, by Δt. The time it takes for a wave to propagate the distance x is given by x/v_{sound}. The time it takes for a wave to propagate the distance $x - \Delta x$ is given by $(x - \Delta x)/v_{\text{sound}}$. But, the onset of the emission of the complete oscillation started a time T earlier than the end of the emission of the complete oscillation. Hence,

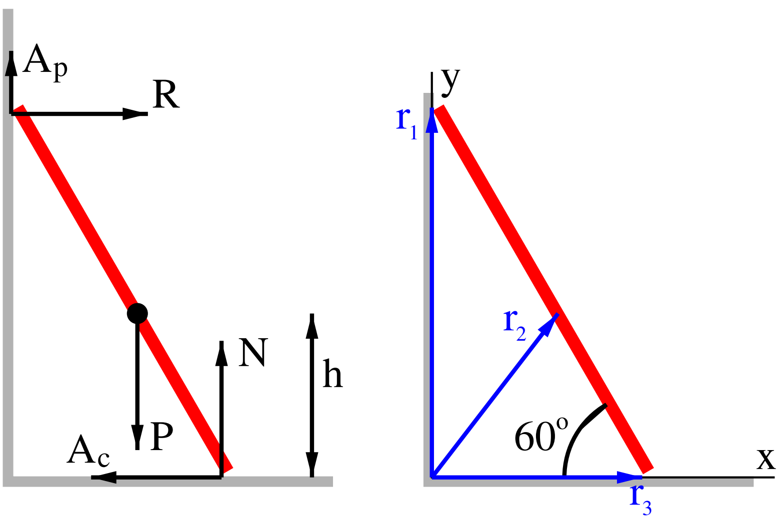
$$\begin{aligned}\Delta t &= T + \frac{x - \Delta x}{v_{\text{sound}}} - \frac{x}{v_{\text{sound}}} = T - \frac{\Delta x}{v_{\text{sound}}} = \frac{1}{f} - \frac{1}{f}\frac{v_{\text{train}}}{v_{\text{sound}}} \\ &= \frac{1}{f}\left(1 - \frac{v_{\text{train}}}{v_{\text{sound}}}\right) = \frac{1}{55 \text{ s}^{-1}}\left(1 - \frac{100 \text{ m/s}}{340 \text{ m/s}}\right) = 0.0128 \text{ s}\end{aligned}$$

c: The person's eardrum makes one complete oscillation in 0.0128 s. Hence, the frequency the person observes equals

$$f_{\text{person}} = \frac{1}{0.0128 \text{ s}} = 78 \text{ Hz}$$

which is a higher pitch, actually $(D^{\#}/E^{b})_2$, than what the train emits.

Exercício 2



In the figure we show the five forces which are involved in this case: The weight of the person $\vec{P}$, the normal reaction force of the floor $\vec{N}$, the friction force between the ladder and the floor $\vec{A}_c$, the normal reaction force of the wall $\vec{R}$ and the friction force between the ladder and the wall $\vec{A}_p$. In the case of point particles all forces apply in the same point. However, in the case of extended

objects the forces have different points of application. In the righthand figure we have indicated the positions where the four forces apply: $\vec{r}_1$ where the normal reaction force of the wall $\vec{R}$ and the friction force between the ladder and the wall $\vec{A}_p$ apply, $\vec{r}_2$ where the weight of the person $\vec{P}$ applies and $\vec{r}_3$ where the normal reaction force of the floor $\vec{N}$ and the friction force between the ladder and the floor $\vec{A}_c$ apply.
We have chosen the xy coordinates as indicated in the righthand figure. The resulting vector representations for the five forces are then given by

$$\vec{P} = \begin{pmatrix} 0 \\ -mg \end{pmatrix} \quad , \quad \vec{N} = \begin{pmatrix} 0 \\ N \end{pmatrix} \quad , \quad \vec{A}_c = \begin{pmatrix} -A_c \\ 0 \end{pmatrix} \quad , \quad \vec{R} = \begin{pmatrix} R \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{A}_p = \begin{pmatrix} 0 \\ A_p \end{pmatrix}$$

Let us denote the height where the person stays on the ladder by h and, furthermore, the length of the ladder by ℓ ($\ell = 5$ m). Then, for the distance from the origin of our coordinate system to the place where the ladder touches the floor, one finds $\ell \cos(60°) = \ell/2$, whereas the ladder touches the wall at a height of $\ell \sin(60°) = \ell\sqrt{3}/2$ Consequently,

$$\vec{r}_1 = \begin{pmatrix} 0 \\ \frac{1}{2}\ell\sqrt{3} \end{pmatrix} \quad , \quad \vec{r}_2 = \begin{pmatrix} \frac{1}{2}\ell - \dfrac{h}{\sqrt{3}} \\ h \end{pmatrix} \quad \text{and} \quad \vec{r}_3 = \begin{pmatrix} \frac{1}{2}\ell \\ 0 \end{pmatrix}$$

When there is equilibrium, then the forces cancel each other (first law of Newton). Hence:

$$0 = \vec{P} + \vec{N} + \vec{A}_c + \vec{R} + \vec{A}_p = \begin{pmatrix} R - A \\ -mg + N + A_p \end{pmatrix}$$

From which we obtain

$$A_c = R \quad \text{and} \quad N + A_p = P = mg$$

Consequently, the forces $\vec{R}$, $\vec{A}_p$, $\vec{N}$ and $\vec{A}_c$ are undetermined.

But, unlike point particles, extended objects can rotate. They do so when a torque is acting on them. In equilibrium, when nothing rotates, there is no torque acting on the ladder. Hence, for the torques of the five forces one has

$$\begin{aligned} 0 &= \vec{r}_1 \times \vec{R} + \vec{r}_1 \times \vec{A}_p + \vec{r}_2 \times \vec{P} + \vec{r}_3 \times \vec{N} + \vec{r}_3 \times \vec{A}_c \\ &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\ell\sqrt{3}R \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\left(\frac{1}{2}\ell - \dfrac{h}{\sqrt{3}}\right) mg \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2}\ell N \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\ell\sqrt{3}A_c - \left(\frac{1}{2}\ell - \dfrac{h}{\sqrt{3}}\right)(N + A_p) + \frac{1}{2}\ell N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\ell\sqrt{3}A_c - \left(\frac{1}{2}\ell - \dfrac{h}{\sqrt{3}}\right) A_p + \dfrac{h}{\sqrt{3}} N \end{pmatrix} \end{aligned}$$

Notice that the torque vectors associated with anticlockwise rotation are in the positive z direction, whereas those associated with clockwise rotation are in the negative z direction.

We find thus:

$$\frac{A_c}{N} + \left(\frac{1}{\sqrt{3}} - \frac{2h}{3\ell}\right)\frac{A_p}{N} = \frac{2h}{3\ell}$$

Furthermore we know that

$$\frac{A_c}{N} \leq \text{(static friction coefficient ladder–floor)} = 0.20$$

and

$$\frac{A_p}{R} \leq \text{(static friction coefficient ladder–wall)} = 0.50$$

Moreover $R = A_c$, hence

$$\frac{A_p}{N} = \frac{A_p}{R}\frac{R}{N} = \frac{A_p}{R}\frac{A_c}{N} \leq (0.50) \times (0.20) = 0.10$$

Consequently, when $\left(\frac{1}{\sqrt{3}} - \frac{2h}{3\ell}\right) \geq 0$,

$$\frac{2h}{3\ell} = \frac{A_c}{N} + \left(\frac{1}{\sqrt{3}} - \frac{2h}{3\ell}\right)\frac{A_p}{N} \leq 0.20 + \left(\frac{1}{\sqrt{3}} - \frac{2h}{3\ell}\right)0.10$$

from which we deduce

$$1.10 \times \frac{2h}{3\ell} \leq 0.20 + \frac{0.10}{\sqrt{3}}$$

and which leads to the equation

$$h \leq \frac{3}{2} \times \frac{0.20 + 0.10/\sqrt{3}}{1.10}\,\ell = 0.351\,\ell = 1.76 \text{ m}.$$

We may moreover verify that indeed $\left(\frac{1}{\sqrt{3}} - \frac{2h}{3\ell}\right) \geq 0$.

Exercício 3

After rolling downhill a distance s the height of the wheel has changed by $h = s\sin(\theta)$, whereas its velocity is given by $v = ds/dt$. Its gain in kinetic energy is related to its loss in gravitational potential energy by

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = E_{\text{kin}} = E_{\text{pot}} = mgh$$

The relation between ω and v is given by

$$v = \omega R \quad \Longleftrightarrow \quad I\omega^2 = \tfrac{1}{2}mR^2\left(\frac{v}{R}\right)^2 = \tfrac{1}{2}mv^2$$

So we have the relation

$$\tfrac{3}{4}mv^2 = E_{\text{kin}} = E_{\text{pot}} = mgh \quad \Longleftrightarrow \quad v^2 = \tfrac{4}{3}gh$$

We take the time derivative on both sides of the above equation (a stands for the acceleration of the wheel).

$$2va = 2v\frac{dv}{dt} = \frac{dv^2}{dt} = \frac{d\left\{\frac{4}{3}gh\right\}}{dt} = \frac{d\left\{\frac{4}{3}gs\sin(\theta)\right\}}{dt} = \tfrac{4}{3}g\sin(\theta)\frac{ds}{dt} = \tfrac{4}{3}g\sin(\theta)v$$

Hence

$$a = \tfrac{2}{3}g\sin(\theta) = \tfrac{2}{3}g\sin(30^\circ) = \tfrac{1}{3}g$$

Exercício 4

a: The temperatures can be obtained from the ideal-gas law:

$$PV = nRT \quad \Longleftrightarrow \quad T_{11} = \frac{V_1 P_1}{nR}$$

with $T_{11} = 300$ K. The number of moles n and the gas constant R are constant during the process. We find then

$$T_{12} = \frac{V_1 P_2}{nR} = \frac{V_1 3P_1}{nR} = 3\frac{V_1 P_1}{nR} = 3T_{11} = 900 \text{ K}$$

$$T_{22} = \frac{V_2 P_2}{nR} = \frac{1.5V_1 3P_1}{nR} = 4.5\frac{V_1 P_1}{nR} = 4.5T_{11} = 1350 \text{ K}$$

and

$$T_{31} = \frac{V_3 P_1}{nR} = \frac{2.5V_1 P_1}{nR} = 2.5\frac{V_1 P_1}{nR} = 2.5T_{11} = 750 \text{ K}$$

b: Here, the difficulty is to evaluate the work done by the engine in the process $(V_2, P_2) \rightarrow (V_3, P_1)$. But, we know that the total effective work $W_{\text{effective}}$ performed by the engine in the complete cycle, is equal to the enclosed area. Consequently,

$$\begin{aligned} W_{\text{effective}} &= (V_2 - V_1)(P_2 - P_1) + \tfrac{1}{2}(V_3 - V_2)(P_2 - P_1) \\ &= (0.5V_1)(2P_1) + \tfrac{1}{2}(1.0V_1)(2P_1) = 2.0P_1V_1 \end{aligned}$$

Furthermore, in the isochoric process $(V_1, P_1) \rightarrow (V_1, P_2)$ no work is performed, since the volume is constant, whereas, in the isobaric process $(V_1, P_2) \rightarrow (V_2, P_2)$ the engine delivered work, $W_{12\rightarrow 22}$, to the exterior, given by

$$W_{12\rightarrow 22} = P_2 (V_2 - V_1) = 3P_1 (0.5V_1) = 1.5P_1V_1$$

and, moreover, in the isobaric process $(V_3, P_1) \rightarrow (V_1, P_1)$ the engine recieved work, $W_{31\rightarrow 11}$, from the exterior, given by

$$W_{31\rightarrow 11} = P_1 (V_1 - V_3) = P_1 (-1.5V_1) = -1.5P_1V_1$$

So, the work $W_{22\rightarrow 31}$ performed by the engine in the process $(V_2, P_2) \rightarrow (V_3, P_1)$ is given by

$$W_{22\rightarrow 31} = W_{\text{effective}} - W_{12\rightarrow 22} - W_{31\rightarrow 11} = 2.0P_1V_1 - 1.5P_1V_1 + 1.5P_1V_1 = 2.0P_1V_1$$

The result $W_{22\rightarrow 31} = 3P_1V_1$ can be obtained directly from the relation of the pressure P in function of the volume V for the process $(V_2, P_2) \rightarrow (V_3, P_1)$. That relation is here given by

$$P(V) = 2P_1\left(3 - \frac{V}{V_1}\right)$$

You can easily verify the above relation by checking $P(V_2) = 2P_1(3-1.5) = 3P_1$ and $P(V_3) = 2P_1(3-2.5) = P_1$, which are indeed the values of the pressure at respectively $V = V_2$ and $V = V_3$ along the line from (V_2, P_2) to (V_3, P_1). Now, the work along the line follows from

$$W_{22\to 31} = \int_{V_2}^{V_3} dV\ P(V) = \int_{1.5V_1}^{2.5V_1} dV\ 2P_1\left(3 - \frac{V}{V_1}\right)$$

The integral is not difficult to be evaluated. One finds

$$\begin{aligned} W_{22\to 31} &= \int_{1.5V_1}^{2.5V_1} dV\ 2P_1\left(3 - \frac{V}{V_1}\right) = 2P_1\left[3V - \frac{V^2}{2V_1}\right]_{1.5V_1}^{2.5V_1} \\ &= 2P_1\left\{\left(3(2.5V_1) - \frac{(2.5V_1)^2}{2V_1}\right) - \left(3(1.5V_1) - \frac{(1.5V_1)^2}{2V_1}\right)\right\} \\ &= 2P_1\left\{7.5 - 3.125 - 4.5 + 1.125\right\} V_1 = 2P_1V_1 \end{aligned}$$

The kinetic energy increases in the process $(V_1, P_1) \to (V_1, P_2)$ because the initial temperature (300 K) is lower than the final temperature (900 K). The difference in kinetic energy K equals

$$K_{11\to 12} = K_{12} - K_{11} = \tfrac{3}{2}V_1P_2 - \tfrac{3}{2}V_1P_1 = \tfrac{3}{2}V_1 3P_1 - \tfrac{3}{2}V_1P_1 = 3.0P_1V_1$$

The heat $Q_{11\to 12}$ which is consumed by the engine in the process $(V_1, P_1) \to (V_1, P_2)$, is thus given by

$$Q_{11\to 12} = W_{11\to 12} + K_{11\to 12} = 0.0 + 3.0P_1V_1 = 3.0P_1V_1$$

The kinetic energy also increases in the process $(V_1, P_2) \to (V_2, P_2)$ because the initial temperature (900 K) is lower than the final temperature (1350 K). The difference in kinetic energy K equals

$$K_{12\to 22} = K_{22} - K_{12} = \tfrac{3}{2}V_2P_2 - \tfrac{3}{2}V_1P_2 = \tfrac{3}{2}1.5V_1 3P_1 - \tfrac{3}{2}V_1 3P_1 = 2.25P_1V_1$$

The heat $Q_{12\to 22}$ which is consumed by the engine in the process $(V_1, P_2) \to (V_2, P_2)$, is thus given by

$$Q_{12\to 22} = W_{12\to 22} + K_{12\to 22} = 1.5P_1V_1 + 2.25P_1V_1 = 3.75P_1V_1$$

The kinetic energy decreases in the process $(V_2, P_2) \to (V_3, P_1)$ because the initial temperature (1350 K) is higher than the final temperature (750 K). The difference in kinetic energy K equals

$$K_{22\to 31} = K_{31} - K_{22} = \tfrac{3}{2}V_3P_1 - \tfrac{3}{2}V_2P_2 = \tfrac{3}{2}2.5V_1P_1 - \tfrac{3}{2}1.5V_1 3P_1 = -3.0P_1V_1$$

The heat $Q_{22\to 31}$ which is released by the engine in the process $(V_2, P_2) \to (V_3, P_1)$, is thus given by

$$Q_{22\to 31} = W_{22\to 31} + K_{22\to 31} = 2.0P_1V_1 - 3.0P_1V_1 = -P_1V_1$$

The kinetic energy also decreases in the process $(V_3, P_1) \to (V_1, P_1)$ because the initial temperature (750 K) is higher than the final temperature (300 K). The difference in kinetic energy K equals

$$K_{31\to 11} = K_{11} - K_{31} = \tfrac{3}{2}V_1P_1 - \tfrac{3}{2}V_3P_1 = \tfrac{3}{2}V_1P_1 - \tfrac{3}{2}2.5V_1P_1 = -2.25P_1V_1$$

Notice

$$K_{11\to12} + K_{12\to22} + K_{22\to31} + K_{31\to11} = 0$$

The heat $Q_{31\to11}$ which is released by the engine in the process $(V_3, P_1) \to (V_1, P_1)$, is thus given by

$$Q_{31\to11} = W_{31\to11} + K_{31\to11} = -1.5P_1V_1 - 2.25P_1V_1 = -3.75P_1V_1$$

c: The total heat consumption by the engine, which occurs during the processes $(V_1, P_1) \to (V_1, P_2)$ and $(V_1, P_2) \to (V_2, P_2)$, equals

$$Q_{\text{total}} = Q_{11\to12} + Q_{12\to22} = 6.75P_1V_1$$

The efficiency is thus readily determined

$$\text{efficiency} = \frac{W_{\text{effective}}}{Q_{\text{total}}} = \frac{2.0P_1V_1}{6.75P_1V_1} = 0.30$$

Exercício 5

a: The weight of the three spheres is given by $P = Mg$, with $g = 9.83$ m/s^2. Hence we obtain

$$P = Mg = (5.245 \text{ kg}) \times \left(9.83 \text{ m/s}^2\right) = 51.56 \text{ N} \quad .$$

The upward buoyant force $F_{\text{Archimedes}}$ exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces. Hence,

$$F_{\text{Archimedes}} = gV\rho_{\text{sea}} \quad .$$

For the three spheres we obtain the volumes

$$V = \tfrac{4}{3}\pi \left\{ \begin{array}{l} (8.42 \times 10^{-2} \text{ m})^3 \\ \\ (10.69 \times 10^{-2} \text{ m})^3 \\ \\ (12.24 \times 10^{-2} \text{ m})^3 \end{array} \right\} = \left\{ \begin{array}{l} 2.500 \times 10^{-3} \text{ m}^3 \\ \\ 5.117 \times 10^{-3} \text{ m}^3 \\ \\ 7.681 \times 10^{-3} \text{ m}^3 \end{array} \right. \quad .$$

Hence, for the buoyant forces on the three spheres when submersed, we obtain

$$F_{\text{Archimedes}} = g \left\{ \begin{array}{l} 2.500 \times 10^{-3} \text{ m}^3 \\ \\ 5.117 \times 10^{-3} \text{ m}^3 \\ \\ 7.681 \times 10^{-3} \text{ m}^3 \end{array} \right\} \rho_{\text{sea}} = \left\{ \begin{array}{l} 25.19 \text{ N} \\ \\ 51.56 \text{ N} \\ \\ 77.39 \text{ N} \end{array} \right. \quad .$$

The buoyant force (25.19 N) on the smaller sphere when submersed is smaller than its weight (51.56 N), hence it will sink. The buoyant force (51.56 N) on the intermediate sphere when submersed is equal to its weight (51.56 N), hence it will not sink. The buoyant force (77.39 N) on the larger sphere when submersed is larger than its weight (51.56 N), hence it will not remain submersed, but it will float with part sticking out of the sea.

b: The pressure of the sea on the smaller sphere at depth d is given by

$$P_{\text{liquid}}(d) = d\rho_{\text{sea}}g$$

Hence,

$$d = \frac{10^7 \text{ Pa}}{\rho_{\text{seawater}}g} = \frac{10^7 \text{ Pa}}{\left(1.025 \times 10^3 \text{ kg/m}^3\right)\left(9.83 \text{ m/s}^2\right)} = 992 \text{ m}$$

So, the sphere will implode at a depth of close to 1 km below the surface of the sea. Notice that the air pressure (10^5 Pa) on the surface of the sea is negligible in this calculus, because it is about 1% of the total pressure of 10^7 Pa.

c: When the sinking smaller sphere reaches the constant terminal velocity one has $dv(t)/dt = 0$. We obtain then

$$-Mg + gV\rho_{\text{sea}} + C_2 r^2 v_{\text{term}}^2 = 0$$

$$\Longleftrightarrow \quad v_{\text{term}} = \sqrt{\frac{Mg - F_{\text{Archimedes}}}{C_2 r^2}} =$$

$$\sqrt{\frac{(51.56 \ \text{ N}) - (25.19 \ \text{ N})}{\left(730 \ \text{ kg/m}^3\right) \times (8.42 \times 10^{-2} \ \text{ m})^2}} = 2.257 \ \text{ m/s} \quad .$$

So, it will take the time

$$t = \frac{992 \text{ m}}{2.257 \ \text{ m/s}} = 440 \ \text{ s} = 7.33 \ \text{ min}$$

to reach a depth of 992 m.