Mestrado Integrado em Engenharia Química
Ano lectivo de 2011/2012, 1º semestre

Exame de Recurso de Física I
3 de Fevereiro de 2012

1. Uma esfera metálica com raio $r = 0.42$ cm e com densidade $\rho_{\text{metal}} = 8.12$ g/cm^3 começa a cair, no instante $t = 0$, dentro de um meio viscoso.
A dinâmica do movimento da esfera é dada por

$$V\rho_{\text{metal}}\frac{dv(t)}{dt} = -gV\rho_{\text{metal}} + gV\rho_{\text{fluido}} - C_1 r v(t) \quad . \tag{1}$$

onde $V = 4\pi r^3/3$ representa o volume da esfera metálica.

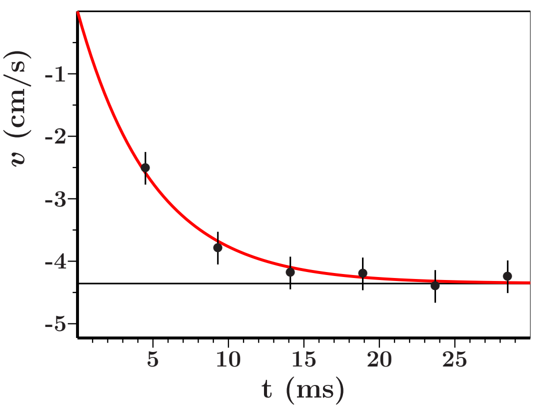
a Interprete cada um dos quatro termos da equação (1).

b Demonstre que a expressão

$$v(t) = v_{\text{term}}\left(e^{-t/T} - 1\right) \tag{2}$$

representa a velocidade $v(t)$ da esfera metálica e determine as relações entre as grandezas v_{term} e T da equação (2) e as grandezas V, ρ_{metal}, ρ_{fluido}, g, C_1 e r da equação (1).

c Os valores obtidos em observações das velocidades da esfera metálica em função do tempo t, estão representados graficamente na figura abaixo.

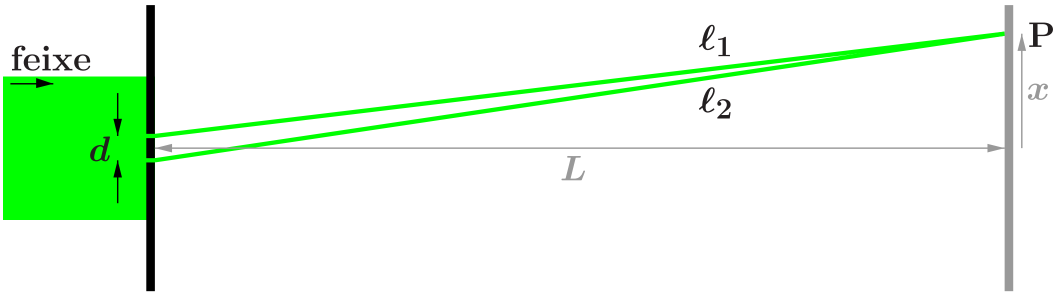


A partir da curva, dada pela expressão

$$v(t) = 4.36\left(e^{-200t} - 1\right) \quad \text{cm/s} \tag{3}$$

(t em segundos), determine o valor do coeficiente de viscosidade C_1 e a densidade do meio viscoso.
Considere a aceleração gravítica $g = 9.8$ m/s^2.

2. Um feixe de luz monocromático passa por duas fendas estreitas e paralelas, que estão a uma distância d, uma da outra, igual a 20 μm. A luz que passa pela fenda superior percorre uma distância ℓ_1 até iluminar um ponto P do ecrã, encontrando-se este a uma distância $L = 5.0$ metros das fendas e paralelo às mesmas. A luz que passa pela fenda inferior percorre uma distância ℓ_2 até ao ponto P que se encontra a uma distância x do centro do ecrã (ver figura).



Considere a intensidade da luz no ponto P dada pela amplitude da função de onda

$$u_{\mathrm{P}}(t) = A \sin(\omega t - k\ell_1) + A \sin(\omega t - k\ell_2) \quad . \tag{4}$$

a Demonstre que

$$u_{\mathrm{P}}(t) = 2A \, \sin(\omega t - k\ell) \, \cos\left(\tfrac{1}{2}k(\ell_2 - \ell_1)\right) \quad , \tag{5}$$

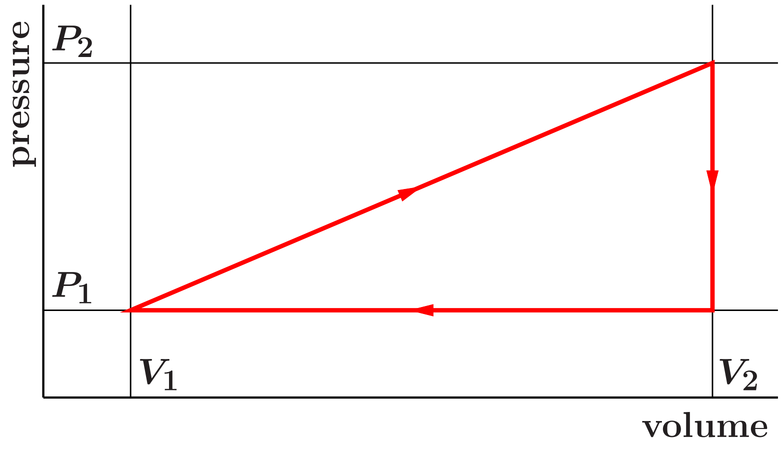
onde $\ell = (\ell_1 + \ell_2)/2$, e interprete os termos $\sin(\omega t - k\ell)$ e $\cos\left(\frac{1}{2}k(\ell_2 - \ell_1)\right)$. Qual a amplitude da expressão (5)?

b Demonstre que para $d \ll x \ll L$ se verifica que $\ell_2 - \ell_1 \approx xd/L$.

c Utilizando a aproximação da alínea anterior, determine a distância x entre o primeiro máximo brilhante observado no ecrã e o centro do ecrã, para luz vermelha ($\lambda = 0.620$ μm) e para luz azul ($\lambda = 0.460$ μm).

3. Considere uma avioneta que, na pista de um aeroporto, descola com uma velocidade de 126 km/h. As asas da avioneta, que têm cada uma uma área de 5.4 m^2, são construídas de tal forma que a velocidade do ar na parte de cima da asa é 40% superior à velocidade da avioneta e na parte de baixo da asa igual à velocidade da avioneta.
Determine a massa máxima que a avioneta juntamente com o piloto e os passageiros pode ter, de modo a ainda ser possível a descolagem.
Considere a aceleração gravítica $g = 9.8$ m/s^2 e a densidade do ar $\rho_{\mathrm{ar}} = 1.29$ kg/m^3.

4. Considere um cilindro de um motor de combustão cujo funcionamento é representado pelo diagrama P-V da figura seguinte.



Considere também as seguintes relações: $V_2 = 2.5V_1$ e $P_2 = 3P_1$.

a Se a temperatura no ponto (V_1,P_1) é igual a 300° K, qual a temperatura nos pontos (V_2,P_2) e (V_2,P_1)?

b Determine a quantidade de calor, em unidades P_1V_1, fornecida ao/pelo combustível na transformação $(V_1, P_1) \rightarrow (V_2, P_2)$.

c Considere a eficácia de um motor definida como o quociente entre o trabalho efectuado e o calor fornecido. Determine a eficácia do motor representado pelo diagrama P-V em causa?

5. Considere uma barra delgada, com largura desprezável e comprimento ℓ. Pelo facto da massa m da barra estar distribuida homogeneamente ao longo do seu comprimento, o momento de inércia da barra é $I = \frac{1}{12}m\ell^2$ para rotações em torno de um eixo perpendicular à barra e que passa pelo seu centro de massa.
Inicialmente a barra está colocada verticalmente sobre um piso perfeitamente liso. No entanto, após uma pequena perturbação, a barra começa a caír.

a Considerando que um extremo da barra desliza livremente sobre o chão e desprezando a resistência do ar, demonstre que o centro de massa da barra cai verticalmente.

b Demonstre que a velocidade com que o outro extremo cai no chão é dada pela expressão $2\sqrt{\frac{3}{4}g\ell}$, onde g representa a aceleração gravítica.

Solutions

Exercício 1

a: $V\rho_{\text{metal}}$ represents the mass m of the metallic sphere and $\frac{dv(t)}{dt}$ its acelleration a. Hence, the term on the lefthand side of equation (1) represents mass×acelleration of the metallic sphere,

$$V\rho_{\text{metal}}\frac{dv(t)}{dt} = ma \quad ,$$

which, by Newton's law, equals the total force acting on the metallic sphere. Consequently, the terms on the righthand side of equation (1) represent the various contributions to the total force which acts on the metallic sphere.
The first term on the righthand side of equation (1)

$$-gV\rho_{\text{metal}} = -mg$$

represents the weight of the metallic sphere, which acts downward.
V represents the volume of the metallic sphere, which is equal to the volume of displaced liquid. $V\rho_{\text{fluido}}$ represents thus the mass of the displaced liquid and $gV\rho_{\text{fluido}}$ the weight of the displaced liquid. Hence, the second term on the righthand side of equation (1)

$$+gV\rho_{\text{fluido}}$$

represents the Arquimedes force on the metallic sphere, which acts upward.
The third term on the righthand side of equation (1)

$$-C_1 rv(t)$$

represents the resistive force of the viscous medium. It is proportional to the radius r and the velocity v of the metallic sphere, which implies that the motion of the metallic sphere is determined by regime I. The velocity of the metallic sphere is downward, hence negative. Therefor, $-C_1 rv(t)$ is positive. Hence, the resistive force of the medium is upward.

b: First, we verify

$$v(t=0) = v_{\text{term}}\left(e^{-0/T} - 1\right) = v_{\text{term}}\left(1-1\right) = 0$$

which indeed agrees with the motion of the metal sphere that starts moving with zero velocity at $t=0$.
Next, we determine the time derivative of expression (2):

$$\begin{aligned}\frac{dv(t)}{dt} &= \frac{d\, v_{\text{term}}\left(e^{-t/T}-1\right)}{dt} = -\frac{v_{\text{term}}}{T}\, e^{-t/T} \\ &= -\frac{v_{\text{term}}}{T}\, e^{-t/T} + \frac{v_{\text{term}}}{T} - \frac{v_{\text{term}}}{T} = -\frac{v_{\text{term}}}{T}\left(e^{-t/T}-1\right) - \frac{v_{\text{term}}}{T} = -\frac{v(t)}{T} - \frac{v_{\text{term}}}{T}\end{aligned}$$

Subsequently, we substitute the result in equation (1):

$$\begin{aligned}
V\rho_{\text{metal}}\left(-\frac{v(t)}{T}-\frac{v_{\text{term}}}{T}\right) &= -gV\rho_{\text{metal}}+gV\rho_{\text{fluido}}-C_1rv(t)\\
-\frac{V\rho_{\text{metal}}}{T}v(t)-\frac{V\rho_{\text{metal}}v_{\text{term}}}{T} &= -gV\rho_{\text{metal}}+gV\rho_{\text{fluido}}-C_1rv(t)\\
\left(C_1r-\frac{V\rho_{\text{metal}}}{T}\right)v(t) &= \frac{V\rho_{\text{metal}}v_{\text{term}}}{T}-gV\rho_{\text{metal}}+gV\rho_{\text{fluido}}
\end{aligned}$$

Since the expression at the lefthand side of the above equation varies with time t, it is different at different instants of time. However, the expression at the righthand side of the above equation does not vary with time t. It is a constant. Consequently, the above equation can only be satisfied for $(0=0)$

$$C_1r-\frac{V\rho_{\text{metal}}}{T}=0 \quad \text{and} \quad \frac{V\rho_{\text{metal}}v_{\text{term}}}{T}-gV\rho_{\text{metal}}+gV\rho_{\text{fluido}}=0$$

$$C_1r=\frac{V\rho_{\text{metal}}}{T} \quad \text{and} \quad \frac{V\rho_{\text{metal}}v_{\text{term}}}{T}=gV\rho_{\text{metal}}-gV\rho_{\text{fluido}}$$

$$T=\frac{V\rho_{\text{metal}}}{C_1r} \quad \text{and} \quad v_{\text{term}}=\frac{gV\left(\rho_{\text{metal}}-\rho_{\text{fluido}}\right)}{V\rho_{\text{metal}}}T$$

$$T=\frac{V\rho_{\text{metal}}}{C_1r} \quad \text{and} \quad v_{\text{term}}=\frac{g\left(\rho_{\text{metal}}-\rho_{\text{fluido}}\right)}{\rho_{\text{metal}}}\frac{V\rho_{\text{metal}}}{C_1r}$$

$$T=\frac{V\rho_{\text{metal}}}{C_1r} \quad \text{and} \quad v_{\text{term}}=\frac{gV\left(\rho_{\text{metal}}-\rho_{\text{fluido}}\right)}{C_1r}$$

c: From the expression $v(t)=4.36\times(\exp(-200t)-1)$ (cm/s), we learn that

$$v_{\text{term}}=4.36\ \text{cm/s} \quad \text{and} \quad T=0.005\ \text{s}$$

Hence,

$$\begin{aligned}
\frac{V\rho_{\text{metal}}}{C_1r} &= 0.005\ \text{s}\\
C_1 &= \frac{V\rho_{\text{metal}}}{r\,(0.005\ \text{s})}=\frac{\frac{4}{3}\pi r^3\rho_{\text{metal}}}{r\,(0.005\ \text{s})}=\frac{\frac{4}{3}\pi r^2\rho_{\text{metal}}}{(0.005\ \text{s})}=\frac{\frac{4}{3}\pi\,(0.42\ \text{cm})^2\left(8.12\ \text{g/cm}^3\right)}{(0.005\ \text{s})}\\
&= \frac{\frac{4}{3}\pi\,(0.42)^2\,(8.12\ \text{g/cm})}{(0.005\ \text{s})}=1200\ \frac{\text{g}}{\text{cm s}}=1200\ \frac{10^{-3}\ \text{kg}}{10^{-2}\ \text{m s}}=120\ \frac{\text{kg}}{\text{ms}}
\end{aligned}$$

And

$$v_{\text{term}}=\frac{gV\left(\rho_{\text{metal}}-\rho_{\text{fluido}}\right)}{C_1r} \iff \rho_{\text{fluido}}=\rho_{\text{metal}}-\frac{C_1rv_{\text{term}}}{gV}$$

Hence,

$$\begin{aligned}
\rho_{\text{fluido}} &= \rho_{\text{metal}} - \frac{C_1 r v_{\text{term}}}{g\frac{4}{3}\pi r^3} = \rho_{\text{metal}} - \frac{C_1 v_{\text{term}}}{g\frac{4}{3}\pi r^2} \\
&= \left(8.12 \text{ g/cm}^3\right) - \frac{(1200 \text{ g/(cm s)})\,(4.36 \text{ cm/s})}{\left(9.8 \text{ m/s}^2\right)\frac{4}{3}\pi\,(0.42 \text{ cm})^2} \\
&= \left(8.12 \text{ g/cm}^3\right) - 723\,\frac{\text{g}}{\text{m cm}^2} = \left(8.12 \text{ g/cm}^3\right) - \left(7.23 \text{ g/cm}^3\right) = 0.89 \text{ g/cm}^3
\end{aligned}$$

Exercício 2

a:

$$-\ell + \tfrac{1}{2}\left(\ell_2 - \ell_1\right) = -\tfrac{1}{2}\left(\ell_2 + \ell_1\right) + \tfrac{1}{2}\left(\ell_2 - \ell_1\right) = -\ell_1$$

and

$$-\ell - \tfrac{1}{2}\left(\ell_2 - \ell_1\right) = -\tfrac{1}{2}\left(\ell_2 + \ell_1\right) - \tfrac{1}{2}\left(\ell_2 - \ell_1\right) = -\ell_2$$

Hence

$$\begin{aligned}
&\sin\left(\omega t - k\ell_1\right) + \sin\left(\omega t - k\ell_2\right) = \\
&= \sin\left(\{\omega t - k\ell\} + \left\{\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right\}\right) + \sin\left(\{\omega t - k\ell\} - \left\{\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right\}\right) \\
&= \sin\left(\omega t - k\ell\right)\,\cos\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right) + \cos\left(\omega t - k\ell\right)\,\sin\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right) \\
&\quad + \sin\left(\omega t - k\ell\right)\,\cos\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right) - \cos\left(\omega t - k\ell\right)\,\sin\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right) \\
&= 2\,\sin\left(\omega t - k\ell\right)\,\cos\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right)
\end{aligned}$$

The $\sin\left(\omega t - k\ell\right)$ term contains the time t. Hence, that term describes the oscillatory motion of light. Those oscillations are very rapid, since

$$f\lambda = c = 3 \times 10^8 \text{ m/s}$$

For visible light (red) one has $\lambda \approx 0.6 \times 10^{-6}$ m and thus

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{0.6 \times 10^{-6} \text{ m}} = 5 \times 10^{14} \text{ Hz}$$

The term $\cos\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right)$ does not vary with time. It is constant in time, but different for each different position. The amplitude of the oscillations at position P is given by

$$2A\,\cos\left(\tfrac{1}{2}k\left(\ell_2 - \ell_1\right)\right)$$

The amplitude varies from zero (minima) when $\frac{1}{2}k\left(\ell_2 - \ell_1\right)$ equals $\pm\frac{1}{2}\pi$, $\pm\frac{3}{2}\pi$, $\pm\frac{5}{2}\pi$, ..., to ± 1 (maxima) when $\frac{1}{2}k\left(\ell_2 - \ell_1\right)$ equals 0 (center of the screen), $\pm\pi$, $\pm 2\pi$, $\pm 3\pi$, ... The intensity of light equals the square of the amplitude.

b: Using the figure of problem (2), it is straightforward to deduce that

$$\ell_2^2 = L^2 + \left(x + \tfrac{1}{2}d\right)^2 \quad \text{and} \quad \ell_1^2 = L^2 + \left(x - \tfrac{1}{2}d\right)^2 \quad .$$

Hence,

$$\left(\ell_2 + \ell_1\right)\left(\ell_2 - \ell_1\right) = \ell_2^2 - \ell_1^2 = \left(x + \tfrac{1}{2}d\right)^2 - \left(x - \tfrac{1}{2}d\right)^2 = 2xd$$

Furthermore, for $d \ll x \ll L$ one reads from the figure

$$\ell_2 + \ell_1 \approx 2L$$

Consequently,

$$\ell_2 - \ell_1 = \frac{2xd}{\ell_2 + \ell_1} \approx \frac{2xd}{2L} = \frac{xd}{L}$$

c: We have the first maximum when

$$\tfrac{1}{2}k\left(\ell_2 - \ell_1\right) = \pm\pi$$

Hence, at the first maximum

$$\tfrac{1}{2}k\frac{xd}{L} \approx \tfrac{1}{2}k\left(\ell_2 - \ell_1\right) = \pm\pi$$

with $k = 2\pi/\lambda$

$$\frac{1}{2}\frac{2\pi}{\lambda}\frac{xd}{L} \approx \pm\pi \quad \Longleftrightarrow \quad x \approx \pm\frac{L\lambda}{d}$$

For red light

$$x \approx \pm\frac{L\lambda}{d} = \pm\frac{(5.0\text{ m})\left(0.62 \times 10^{-6}\text{ m}\right)}{20 \times 10^{-6}\text{ m}} = \pm 0.155\text{ m}$$

For blue light

$$x \approx \pm\frac{L\lambda}{d} = \pm\frac{(5.0\text{ m})\left(0.46 \times 10^{-6}\text{ m}\right)}{20 \times 10^{-6}\text{ m}} = \pm 0.115\text{ m}$$

Exercício 3

Bernoulli:

$$p_{\text{upper part wing}} + \tfrac{1}{2}\rho_{\text{ar}}v^2_{\text{upper part wing}} = p_{\text{lower part wing}} + \tfrac{1}{2}\rho_{\text{ar}}v^2_{\text{lower part wing}}$$

and, moreover,

$$v_{\text{avioneta}} = 126\text{ km/h} = 35\text{ m/s}$$

Hence

$$
\begin{aligned}
& p_{\text{lower part wing}} - p_{\text{upper part wing}} = \\
&= \tfrac{1}{2}\rho_{\text{ar}} v^2_{\text{upper part wing}} - \tfrac{1}{2}\rho_{\text{ar}} v^2_{\text{lower part wing}} \\
&= \tfrac{1}{2}\rho_{\text{ar}} \left(v^2_{\text{upper part wing}} - v^2_{\text{lower part wing}}\right) \\
&= \tfrac{1}{2}\rho_{\text{ar}} \left(v_{\text{upper part wing}} + v_{\text{lower part wing}}\right)\left(v_{\text{upper part wing}} - v_{\text{lower part wing}}\right) \\
&= \tfrac{1}{2}\rho_{\text{ar}} \left(1.4v_{\text{avioneta}} + v_{\text{avioneta}}\right)\left(1.4v_{\text{avioneta}} - v_{\text{avioneta}}\right) \\
&= \tfrac{1}{2}\rho_{\text{ar}} \left(1.4v_{\text{avioneta}} + v_{\text{avioneta}}\right)\left(1.4v_{\text{avioneta}} - v_{\text{avioneta}}\right) \\
&= \tfrac{1}{2}\rho_{\text{ar}}\ 2.4 \times 0.4\ v^2_{\text{avioneta}} = 0.48\rho_{\text{ar}} v^2_{\text{avioneta}} \\
&= 0.48\left(1.29\ \text{kg/m}^3\right)(35\ \text{m/s})^2 = 759\ \frac{\text{kg m}^2}{\text{m}^3\text{s}^2} = 759\ \text{N/m}^2
\end{aligned}
$$

The total lift force on the airplane is equal to

$$\text{lift} = (\text{pressure difference}) \times (\text{area of the wings})$$

We obtain

$$\text{lift} = \left(759\ \text{N/m}^2\right) 2\left(5.4\ \text{m}^2\right) = 8192\ \text{N}$$

For the maximum possible mass of the airplane we find

$$m = \frac{\text{lift}}{g} = \frac{8192\ \text{N}}{9.8\ \text{m/s}^2} = 836\ \text{kg}$$

Exercício 4

a: The temperatures can be obtained from the ideal-gas law:

$$PV = nRT \quad \Longleftrightarrow \quad T_{11} = \frac{V_1 P_1}{nR}$$

with $T_{11} = 300$ K. The number of moles n and the gas constant R are constant during the process. We find then

$$T_{22} = \frac{V_2 P_2}{nR} = \frac{2.5V_1 3P_1}{nR} = 7.5\frac{V_1 P_1}{nR} = 7.5T_{11} = 2250\ \text{K}$$

and

$$T_{21} = \frac{V_2 P_1}{nR} = \frac{2.5V_1 P_1}{nR} = 2.5\frac{V_1 P_1}{nR} = 2.5T_{11} = 750\ \text{K}$$

b: Here, the difficulty is to evaluate the work done by the engine. But, we know that the total effective work $W_{\text{effective}}$ performed by the engine in the complete cycle, is equal to the enclosed area. Consequently,

$$W_{\text{effective}} = \tfrac{1}{2}\left(V_2 - V_1\right)\left(P_2 - P_1\right) = \tfrac{1}{2}\left(1.5V_1\right)\left(2P_1\right) = 1.5P_1V_1$$

Furthermore, in the isochoric process $(V_2, P_2) \to (V_2, P_1)$ no work is performed, since the volume is constant, whereas, in the isobaric process $(V_2, P_1) \to (V_1, P_1)$ the engine recieved work $W_{21\to 11}$ from the exterior, given by

$$W_{21\to 11} = P_1 (V_1 - V_2) = P_1 (-1.5V_1) = -1.5P_1V_1$$

So, the work $W_{11\to 22}$ performed by the engine in the process $(V_1, P_1) \to (V_2, P_2)$ is given by

$$W_{11\to 22} = W_{\text{effective}} - W_{21\to 11} = 3P_1V_1$$

The kinetic energy increases in the process $(V_1, P_1) \to (V_2, P_2)$ because the initial temperature (300 K) is lower than the final temperature (2250 K). The difference in kinetic energy K equals

$$K_{11\to 22} = K_{22} - K_{11} = \tfrac{3}{2}V_2P_2 - \tfrac{3}{2}V_1P_1 = \tfrac{3}{2}2.5V_1 3P_1 - \tfrac{3}{2}V_1P_1 = 9.75P_1V_1$$

The heat $Q_{11\to 22}$ which is consumed by the engine in the process $(V_1, P_1) \to (V_2, P_2)$, is thus given by

$$Q_{11\to 22} = W_{11\to 22} + K_{11\to 22} = 3P_1V_1 + 9.75P_1V_1 = 12.75P_1V_1$$

c: In the isochoric process $(V_2, P_2) \to (V_2, P_1)$ the engine does not consume heat, but releases heat, given by

$$Q_{22\to 21} = K_{22\to 21} = K_{21} - K_{22} = \tfrac{3}{2}V_2P_1 - \tfrac{3}{2}V_2P_2 = \tfrac{3}{2}2.5V_1P_1 - \tfrac{3}{2}2.5V_1 3P_1 = -7.5P_1V_1$$

Furthermore, in the isobaric process $(V_2, P_1) \to (V_1, P_1)$ the engine does also not consume heat, but releases heat, given by

$$Q_{21\to 11} = K_{21\to 11} = K_{11} - K_{21} = \tfrac{3}{2}V_1P_1 - \tfrac{3}{2}V_2P_1 = \tfrac{3}{2}V_1P_1 - \tfrac{3}{2}2.5V_1P_1 = -2.25P_1V_1$$

Hence, the total heat consumption by the engine, which only occurs during the process $(V_1, P_1) \to (V_2, P_2)$, equals

$$Q_{\text{total}} = Q_{11\to 22} = 12.75P_1V_1$$

The efficiency is thus readily determined

$$\text{efficiency} = \frac{W_{\text{effective}}}{Q_{\text{total}}} = \frac{1.5P_1V_1}{12.75P_1V_1} = 0.12$$

Some extra information (not necessary for the exam):

The result $W_{11\to 22} = 3P_1V_1$ can be obtained directly from the relation of the pressure P in function of the volume V for the process $(V_1, P_1) \to (V_2, P_2)$. That relation is here given by

$$P(V) = \frac{P_1}{3}\left(4\frac{V}{V_1} - 1\right)$$

You can easily verify the above relation by checking $P(V_1) = P_1(4-1)/3 = P_1$ and $P(V_2) = P_1(4V_2/V_1 - 1)/3 = P_1(4 \times 2.5 - 1)/3 = 3P_1 = P_2$, which are indeed the values of the pressure at respectively $V = V_1$ and $V = V_2$ along the line from (V_1, P_1) to (V_2, P_2). Now, the work along the line follows from

$$W_{11\to 22} = \int_{V_1}^{V_2} dV\ P(V) = \int_{V_1}^{2.5V_1} dV\ \frac{P_1}{3}\left(4\frac{V}{V_1} - 1\right)$$

The integral is not difficult to be evaluated. One finds

$$\begin{aligned} W_{11\to 22} &= \int_{V_1}^{2.5V_1} dV\, \frac{P_1}{3}\left(4\frac{V}{V_1}-1\right) = \frac{P_1}{3}\left[2\frac{V^2}{V_1}-V\right]_{V_1}^{2.5V_1} \\ &= \frac{P_1}{3}\left\{\left(2\frac{(2.5V_1)^2}{V_1}-2.5V_1\right)-\left(2\frac{(V_1)^2}{V_1}-V_1\right)\right\} \\ &= \frac{P_1}{3}\left\{12.5-2.5-2+1\right\}V_1 = 3P_1V_1 \end{aligned}$$

Exercício 5

a: Since there are no horizontal forces acting on the bar, its center of mass does not move in the horizontal direction. Consequently, it can only move in the vertical direction.

b: The total kinetic energy of the bar is given by the velocity v of the center of mass and by the angular velocity ω of the rotation of the bar arond the center of mass:

$$E_{\text{kinetic}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Since the mass is homogeneously distributed along the bar, the center of mass of the bar is at $\frac{1}{2}\ell$ from the place where one end of the bar is on the floor. Consequently, the relation between ω and v is given by

$$v = \tfrac{1}{2}\ell\omega$$

Hence,

$$I\omega^2 = \tfrac{1}{12}m\ell^2\left(\frac{v}{\frac{1}{2}\ell}\right)^2 = \tfrac{1}{3}mv^2$$

For the kinetic energy we obtain then

$$E_{\text{kinetic}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{6}mv^2 = \tfrac{2}{3}mv^2$$

Before the bar started falling it had a gravitational potential given by

$$E_{\text{gravity}} = mgh = mg\tfrac{1}{2}\ell$$

That energy is all converted into kinetic energy just before the bar touches the floor. Hence,

$$\tfrac{2}{3}mv^2 = mg\tfrac{1}{2}\ell$$

from which we solve

$$v^2 = \tfrac{3}{2}\tfrac{1}{2}g\ell = \tfrac{3}{4}g\ell \quad\Longleftrightarrow\quad v = \sqrt{\tfrac{3}{4}g\ell}$$

The vertical velocity of the upper end of the bar is twice the velocity of the center of mass of the bar, because it is at twice the distance from the end which is on the floor during the motion.