

**NON-EXOTIC  
MESON-MESON SCATTERING  
DOMINATED BY  
S-CHANNEL  
 $Q\bar{Q}$  CONFINEMENT STATES**

**and**

**CLASSIFICATION OF  
THE LIGHT SCALAR MESONS**

prepared for

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coupled channel model  
for  
elastic and inelastic  
 $PP$ ,  $PV$  and  $VV$  scattering

$P$  = pseudoscalar meson

$V$  = vector meson

Example:

$K\pi$   $S$ -wave scattering  
involves 9 channels:

$S$ -wave ( $\ell = 0$  and  $s = 0$ ) :

$K\pi$ ,  $K\eta$ ,  $K\eta'$ ,  $K^*\rho$ ,  $K^*\omega$  and  $K^*\phi$ ,

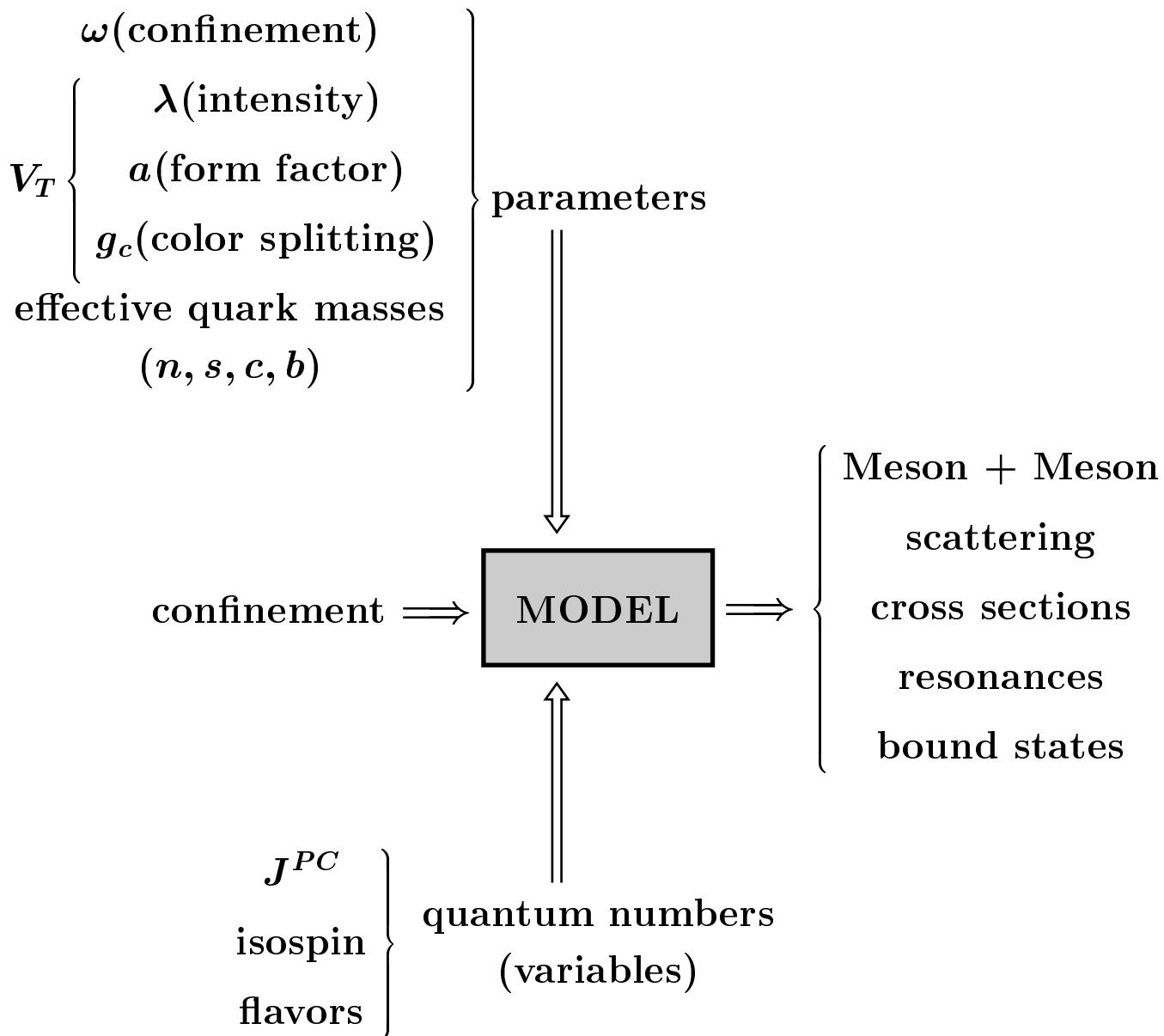
$D$ -wave ( $\ell = 2$  and  $s = 2$ ) :

$K^*\rho$ ,  $K^*\omega$  and  $K^*\phi$

all coupled through

nonstrange-strange

$q\bar{q}$  confinement states



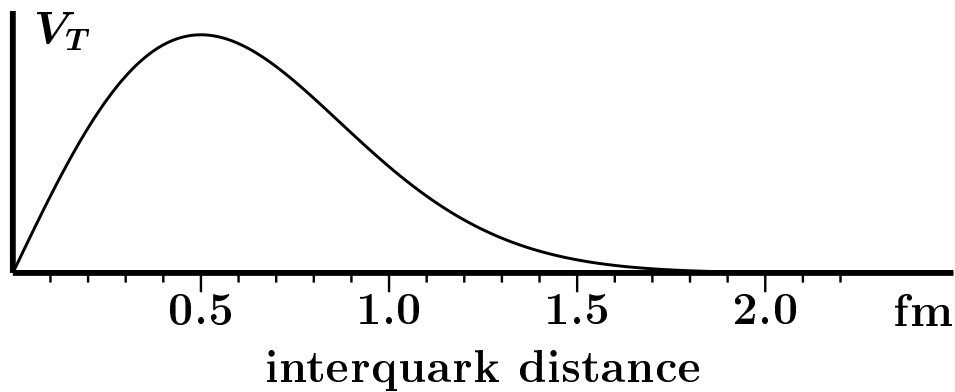
# The relevant parameters of the model

$$\omega = 0.19 \text{ GeV}$$

masses:

$m_n$	$m_s$	$m_c$	$m_b$	
0.406	0.508	1.562	4.724	GeV

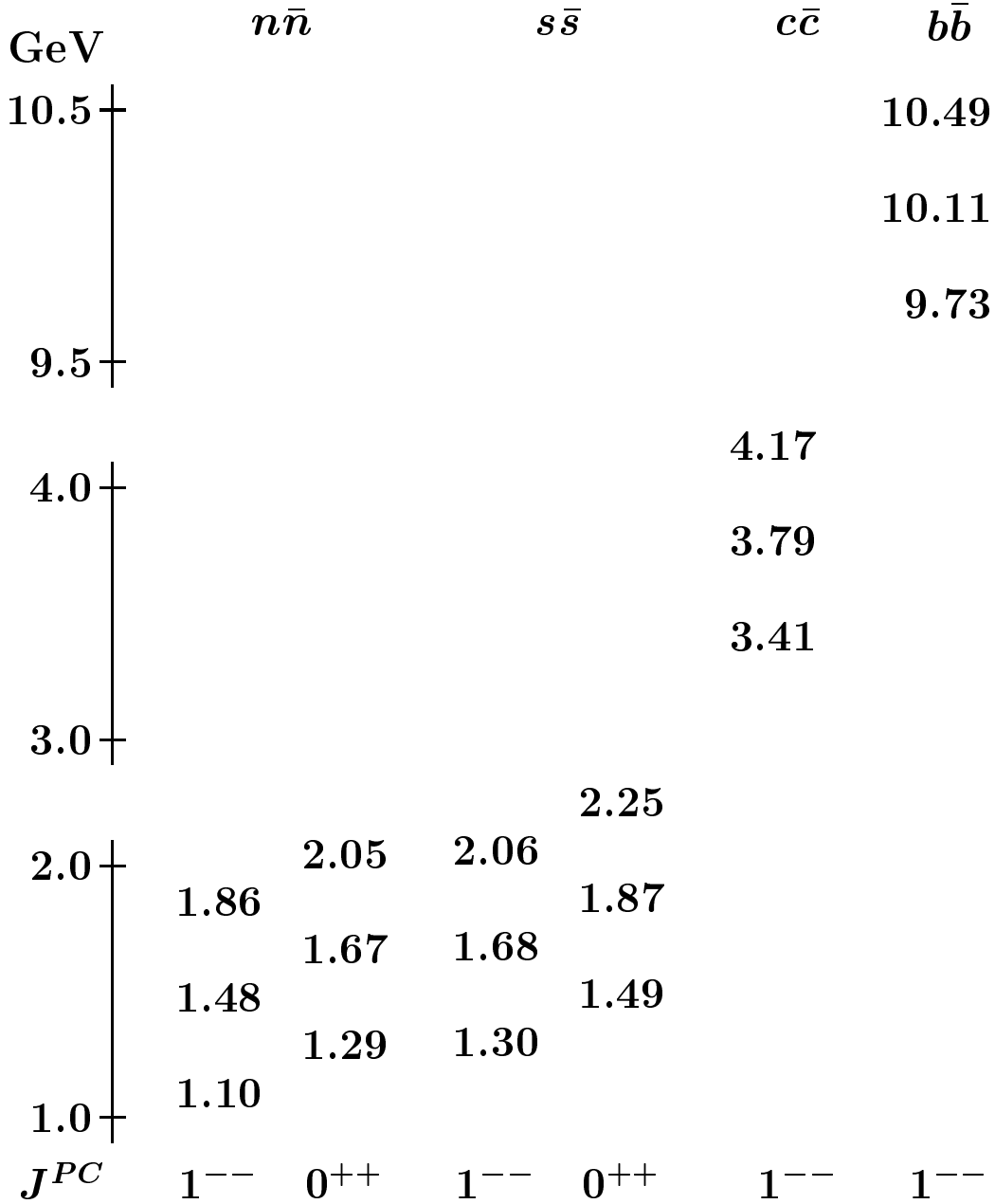
${}^3P_0$  transition mechanism



functional dependence on  
 $J^{PC}$ , isospin and flavors

# confinement spectrum (the bare masses)

$$E_{HO} = \omega \left( 2n + \ell_c + \frac{3}{2} \right) + m_q + m_{\bar{q}}$$



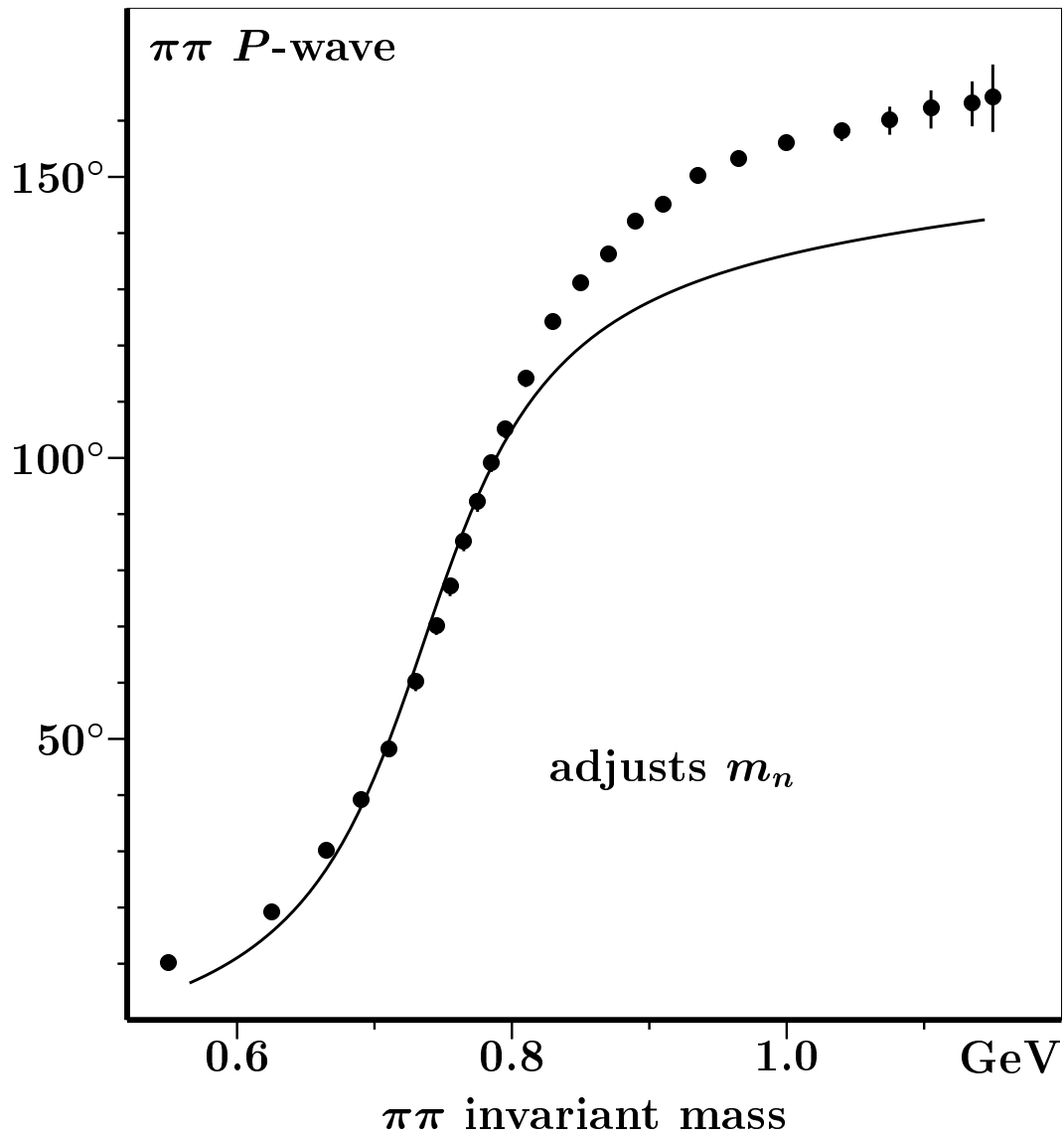
$$J^{PC} = 1^{--} \text{ for } c\bar{c} \text{ and } b\bar{b}$$

Mass GeV	charmonium		beautonium	
	theor	exp	theor	exp
$1^3S_1$	3.08	3.10	9.50	9.46
$2^3S_1$	3.66	3.69	10.00	10.02
$1^3D_1$	3.80	3.77	10.14	...
$3^3S_1$	4.04	4.04	10.39	10.36
$2^3D_1$	4.14	4.16	10.48	...
$4^3S_1$	4.42	4.42	10.77	10.58
$3^3D_1$	...	...	10.86	10.87
$5^3S_1$	...	...	11.15	11.02

comparison to experiment

Phys. Rev. D27,1527 (1983)

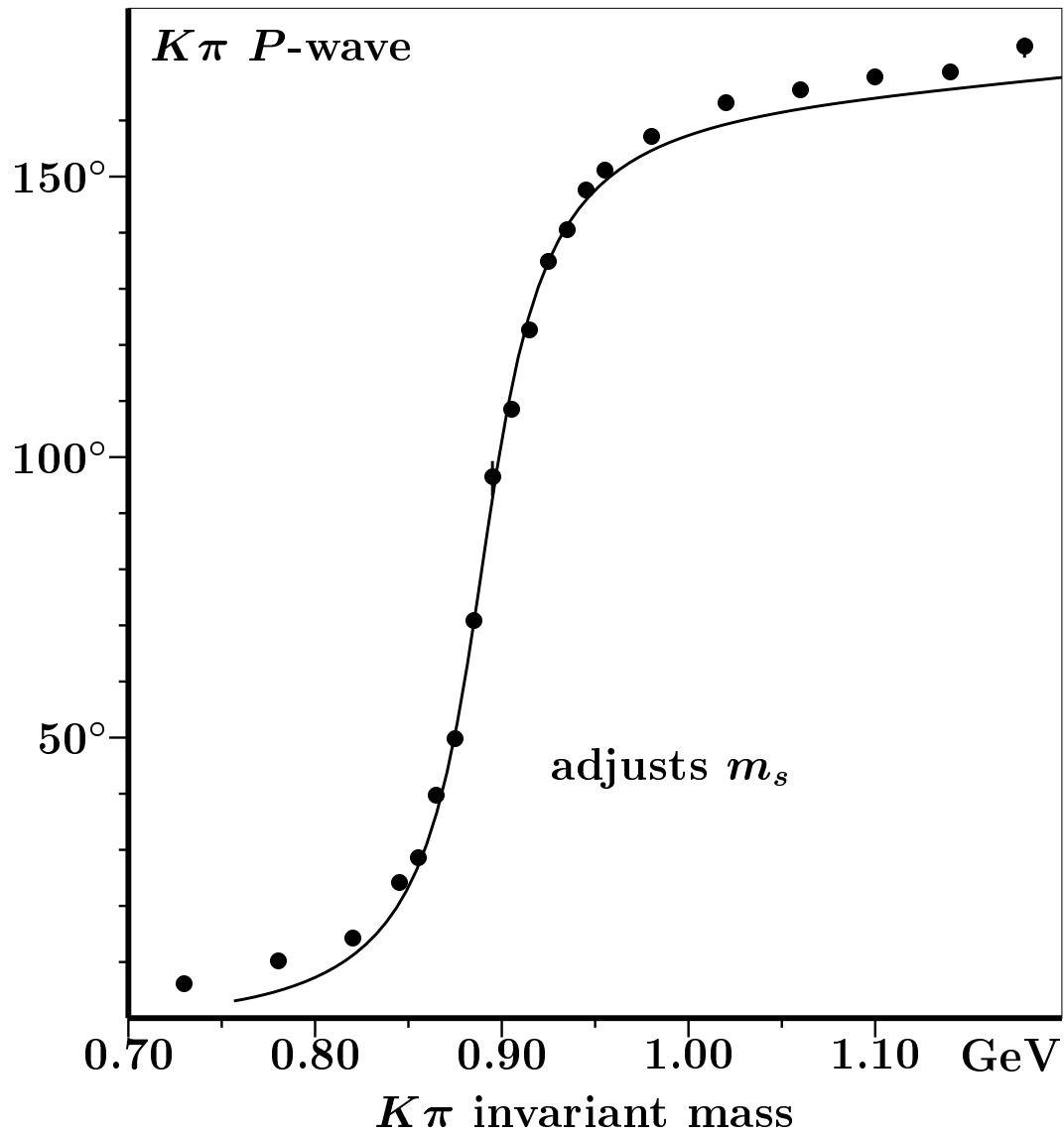
this adjusts:  
 transition potential(s)  
 $\omega$  (confinement)  
 $m_c$  and  $m_b$



$\pi\pi$  elastic  $P$ -wave phase shifts. The data ( $\bullet$ ) are taken from [?]. The model result is the solid line.

## References

- [1] S.D. Protopopescu et al. Phys. Rev. D7,1279 (1973).



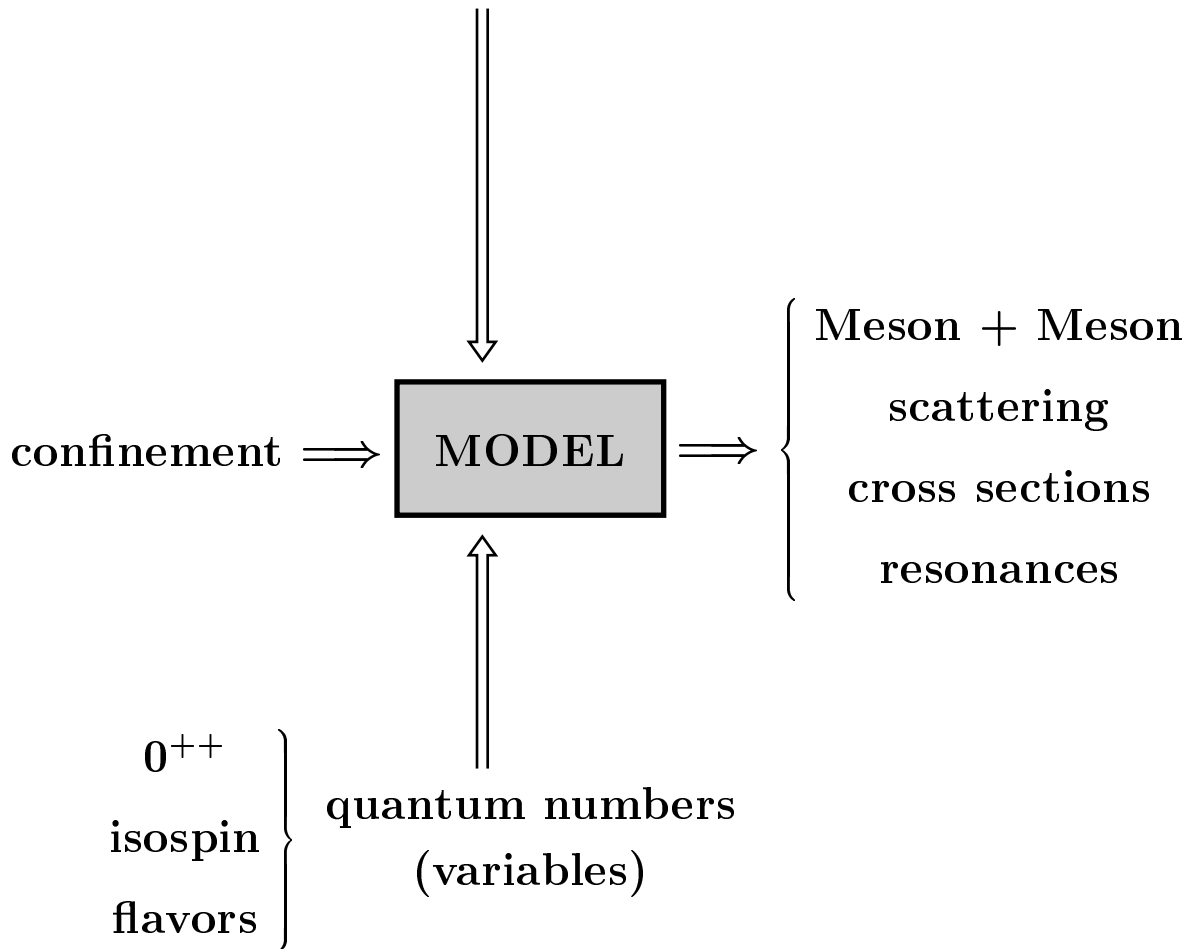
Phase shifts for  $K\pi$  elastic  $P$ -wave scattering. Data are taken from [?]. The model result is the solid line.

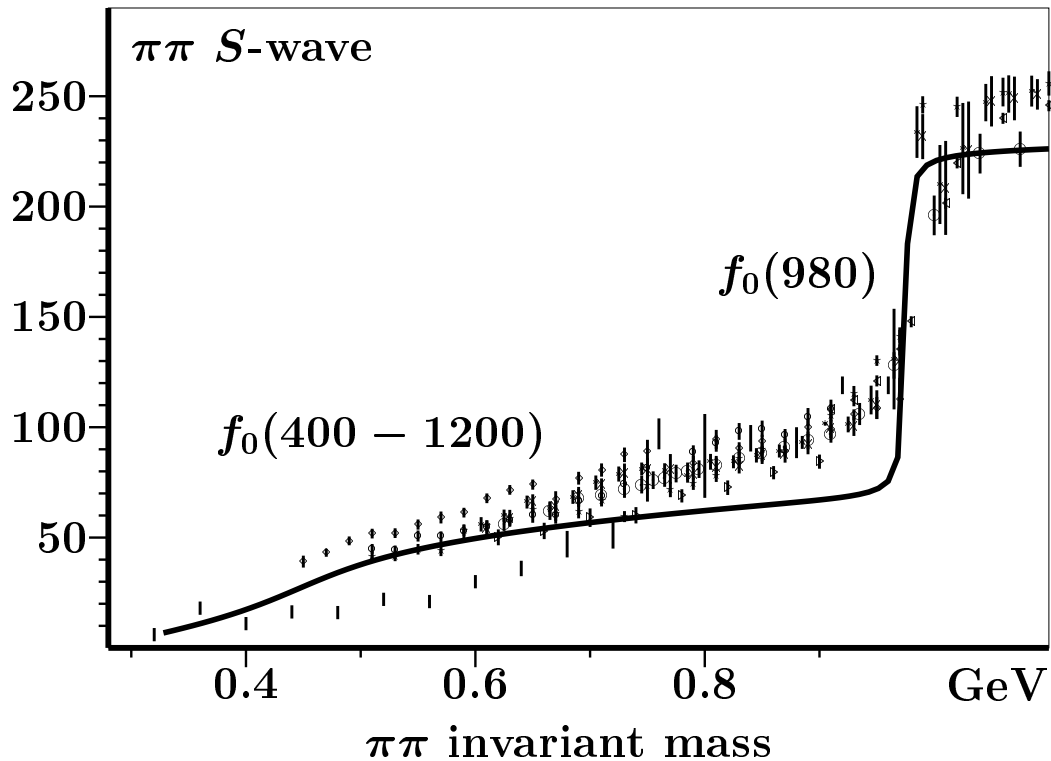
## References

- [1] P. Estabrooks et al. Nucl. Phys. B133,490 (1978).



all parameters fixed

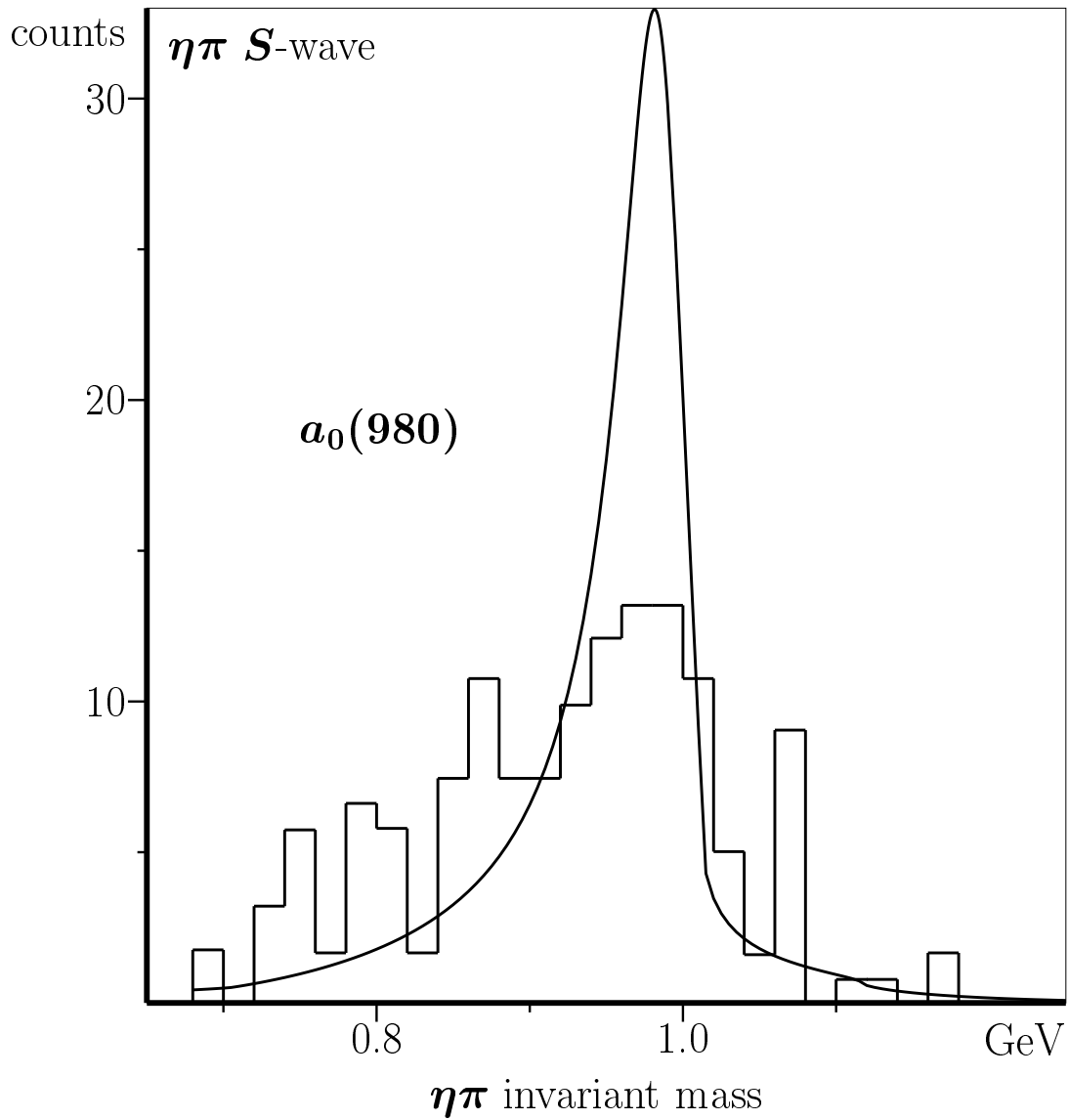




**Remember:**  
 $0^{++}$  bare ground state  
 at 1.29 GeV

## References

- [1] S.D. Protopopescu et al. Phys. Rev. **D7**,1279 (1973).
- [2] B. Hyams et al. Nucl. Phys. **B64**,134 (1973); Nucl. Phys. **B100**,205 (1975).
- [3] G. Grayer et al. Nucl. Phys. **B75**,189 (1974).
- [4] P. Estabrooks and A.D. Martin. Nucl. Phys. **B79**,301 (1974).
- [5] N.N. Biswas et al. Phys. Rev. Lett. **47**,1378 (1981).



**Remember:**  
 $0^{++}$  bare ground state  
 at 1.29 GeV

## References

- [1] M.J. Corden et al. Nucl. Phys. **B144**,253 (1978).

Generalisation for  
arbitrary confinement spectrum  
(e.g. lattice spectrum)

in a simplified version  
of the coupled channel model  
limited to one channel

extension to  
many coupled channels  
and

${}^3P_0$  transitions  
is straightforward

$$\begin{cases} (\mathbf{E} - \mathbf{H}_c) \psi_c(\vec{r}) = \mathbf{V}_t \psi_f(\vec{r}) \\ (\mathbf{E} - \mathbf{H}_f) \psi_f(\vec{r}) = [\mathbf{V}_t]^T \psi_c(\vec{r}) \end{cases}$$

Elimination of confinement sector  $\psi_c(\vec{r})$ :

$$\begin{aligned} (\mathbf{E} - \mathbf{H}_f) \psi_f(\vec{r}) &= \\ &= [\mathbf{V}_t]^T (\mathbf{E} - \mathbf{H}_c)^{-1} \mathbf{V}_t \psi_f(\vec{r}) \end{aligned}$$

$$\Downarrow \text{JPC}$$

$$\cotg(\delta_\ell(p)) = \frac{n_\ell(pa)}{j_\ell(pa)} +$$

$$- \left[ 2\lambda^2 \mu pa j_\ell^2(pa) \sum_{n=0}^{\infty} \frac{|\mathcal{F}_{n\ell_c}(a)|^2}{E - E_{n\ell_c}} \right]^{-1}$$

## $K\pi$ elastic scattering

$$H_f = -\frac{1}{2} \mu^{-1} \nabla_r^2 + M_K + M_\pi$$

$$V_t = \frac{\lambda}{a^{3/2}} \delta(r - a)$$

$$\lambda^2 \sum_{n=0}^{\infty} \frac{|\mathcal{F}_{nl_c}(a)|^2}{E - E_{nl_c}} \approx \lambda^2 \left( \sum_{n=0}^N \frac{B_{nl_c}}{E - E_{nl_c}} - \mathbf{1} \right)$$

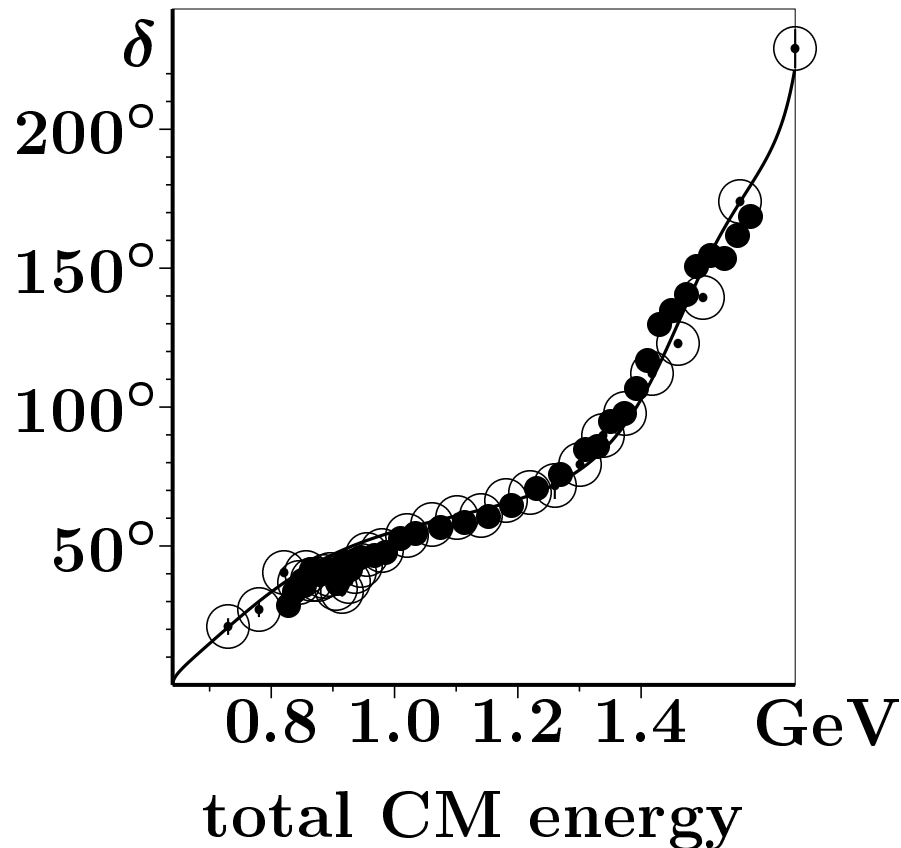
with a redefinition of  $\lambda$

# $K\pi$

Elastic  $I = \frac{1}{2}$   $P$ -wave scattering

$$\lambda = 0.75 \text{ GeV}^{-3/2} \text{ and } a = 5 \text{ GeV}^{-1}$$

$$\sum_{n=0}^{\infty} \frac{|\mathcal{F}_{n,0}(a)|^2}{E - E_{n,0}} \approx \left( \frac{0.5}{E - 0.945} - 1 \right) \text{ GeV}^2$$



confinement sector

lowest  $u\bar{s}$   $J^P = 1^-$  state at

$$E_{0,0} = 0.945 \text{ GeV}$$

scattering singularity can be searched  
for numerically from the analytic  
expression for the cotangent of the  
phase shift  
and is found at

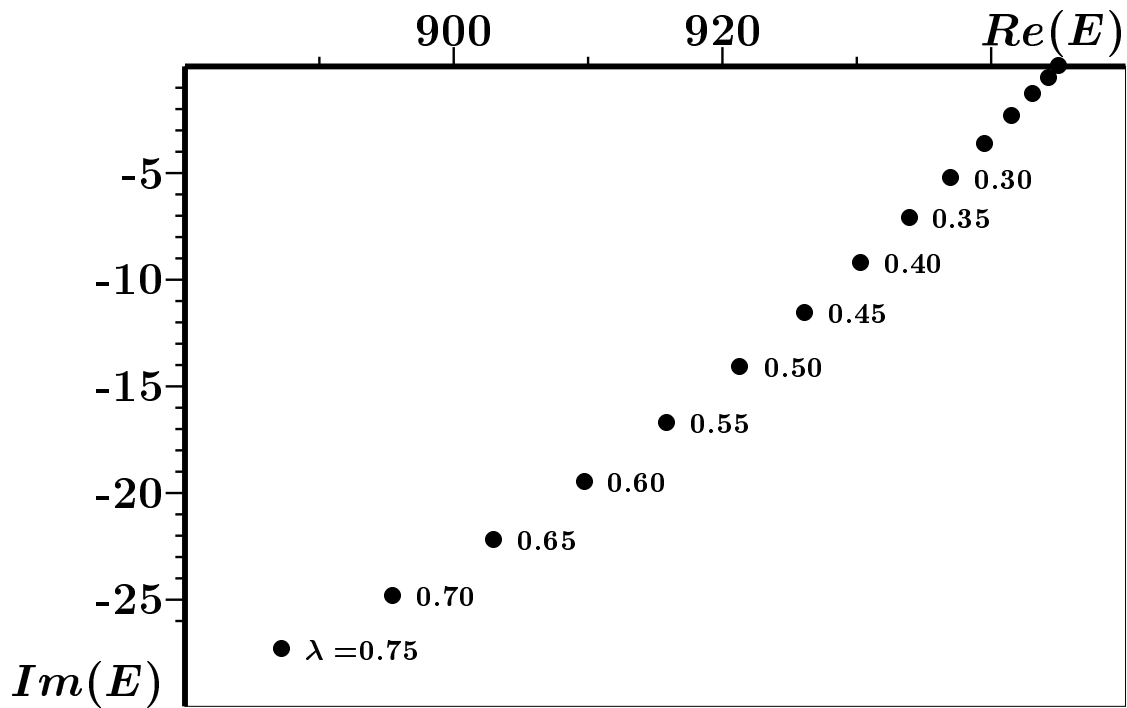
$$0.887 - 0.027i \text{ GeV}$$



# Complex-energy singularities of the $S$ -matrix as function of $\lambda$

The point on the real axis corresponds to the bare state ( $\lambda = 0$ )

Units are in MeV



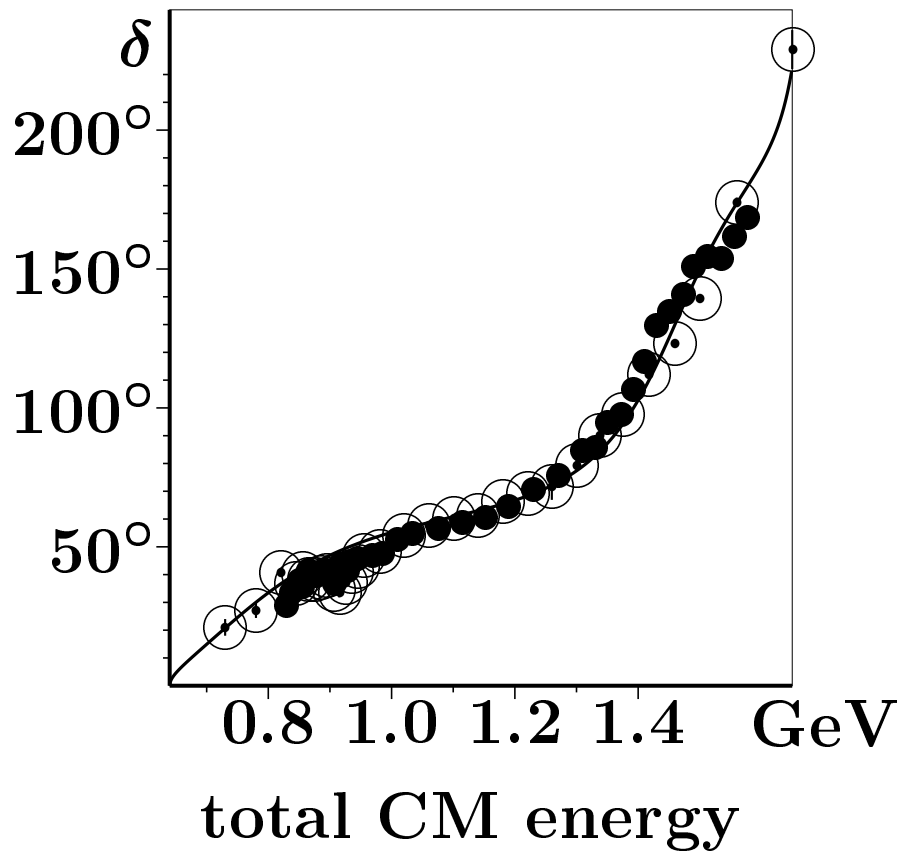
# $K\pi$

Elastic  $I = \frac{1}{2}$   $S$ -wave scattering

$$\lambda = 0.75 \text{ GeV}^{-3/2} \text{ and } a = 3.2 \text{ GeV}^{-1}$$

$$\sum_{n=0}^{\infty} \frac{|\mathcal{F}_{n,1}(a)|^2}{E - E_{n,1}} \approx$$

$$\left( \frac{1.0}{E - 1.31} + \frac{0.2}{E - 1.65} - 1 \right) \text{ GeV}^2$$



confinement sector

lowest bare  $u\bar{s}$   $J^P = 0^+$  states at  
 $E_{0,1} = 1.31$  GeV, and  $E_{1,1} = 1.65$  GeV

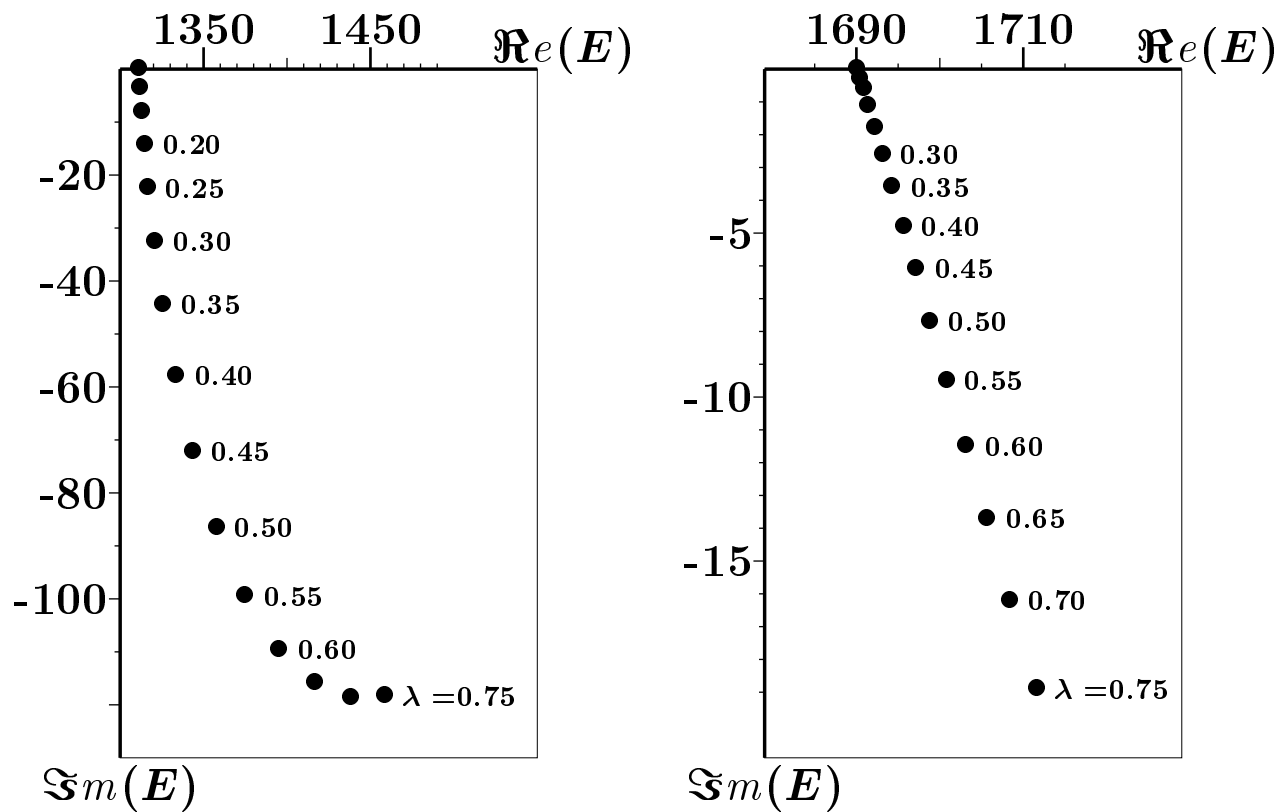
scattering singularities numerically  
 are found at

$1460 - 114i$  MeV,  $1673 - 24i$  MeV,  
 and ...

# Complex-energy singularities of the $S$ -matrix as function of $\lambda$

The point on the real axis corresponds to the bare state ( $\lambda = 0$ )

Units are in MeV



Notice nonperturbative behaviour of lower singularity

and a singularity at

$713 - 227i$  MeV

in

E. van Beveren, T. A. Rijken,  
K. Metzger, C. Dullemond, G. Rupp,  
and J. E. Ribeiro, Zeit. Phys. C30, 615  
(1986)

found at

$727 - 263i$  MeV

many more channels

full transition potential

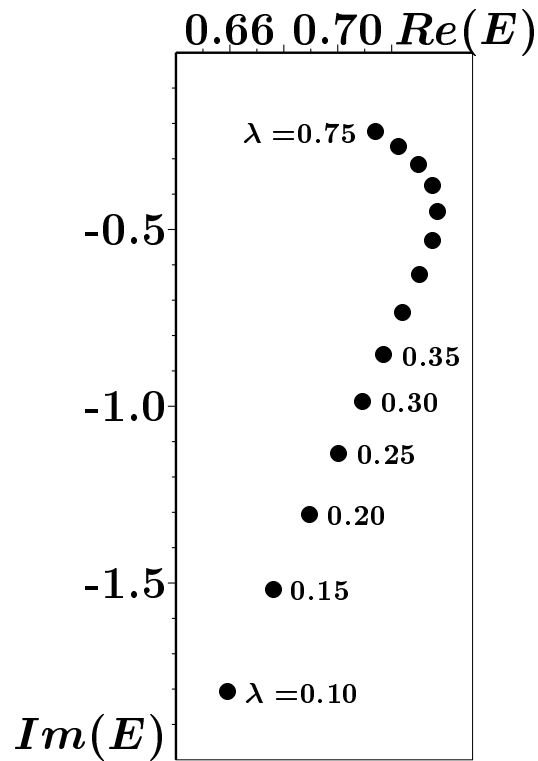
harmonic oscillator confinement

no free parameters

# Complex-energy singularities of the $S$ -matrix as function of $\lambda$

Singularity disappears in background for  $\lambda = 0$

Units are in GeV

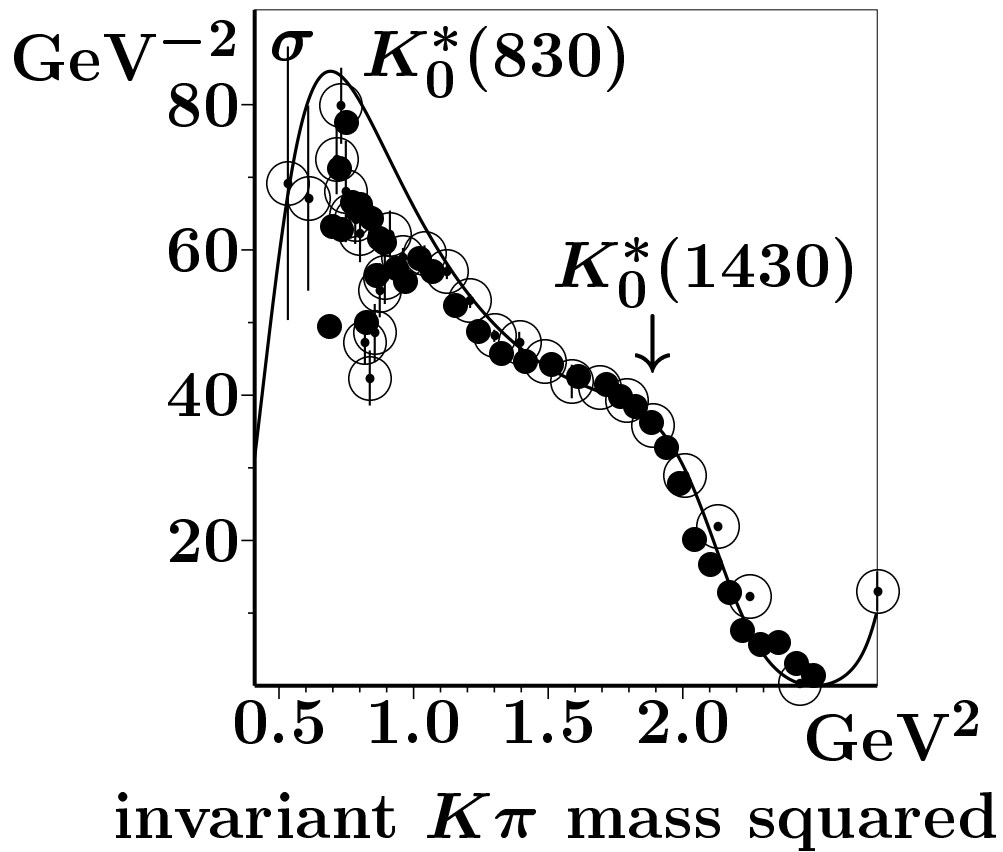


$K_0^*(727)$  completes nonet  
of light scalar mesons

Isospin	pole position (MeV)
$I = 1$	$968-28i$
$I = \frac{1}{2}$	$727-263i$
$I = 0$	$470-208i$ and $994-17i$

forms

THE nonet of  
the lowest lying singularities  
of the scattering matrix  
for  $J^{PC} = 0^{++}$  states



peak signal at 830 MeV

width of some 500 MeV

preliminary result E791 collaboration



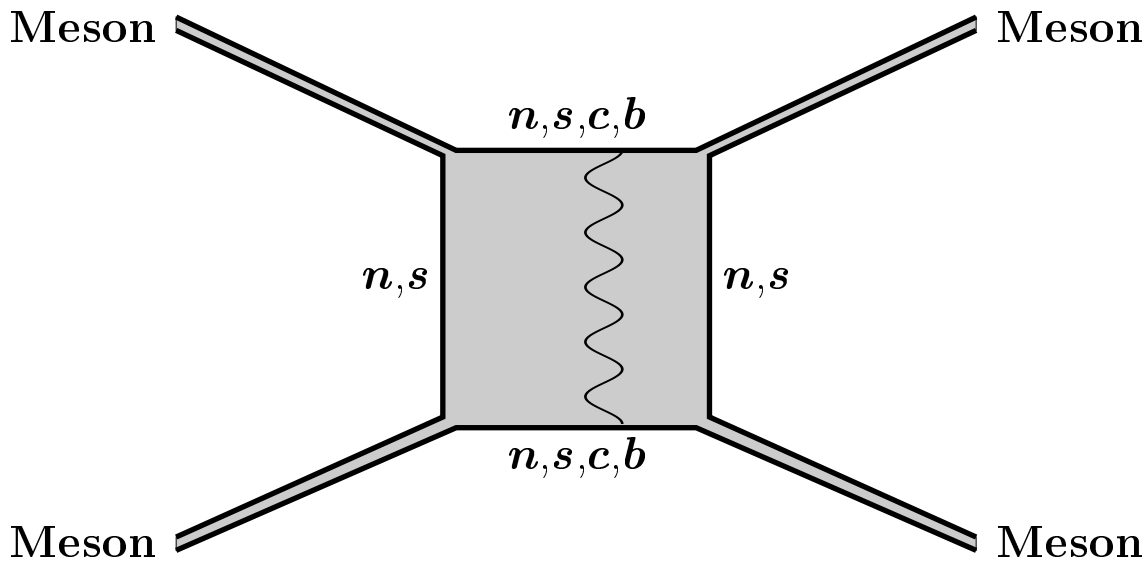
# Understanding the scalar meson nonet?

N.A. Törnqvist, Z.Phys. C68, 647 (1995)

N.A. Törnqvist and M. Roos, Phys.Rev.Lett. 76, 1575 (1996)

## Comparison

	Törnqvist/and Roos	Beveren et al.
model details	unitarisation coupled channels ad hoc bare states	unitarisation coupled channels confinement
date	1995/1996	1986
nonet	$K_0^*(727)$ missing	full
heavy quarks	no relation	in the same model
decay products	pseudoscalars	pseudoscalars and vectors
flavour symmetry	broken	full
parameters	6	none, after fitting to $J^{PC} = 0^{-+}$ and $1^{--}$
higher scalars	mixed up with light scalars	different nonet stemming from confinement
$f_0(1370)$	$s\bar{s}$	$n\bar{n}$
citations (Spire)	125/159	52



## References

- [1] Eef van Beveren, and George Rupp, [hep-ex/0106077].
- [2] Eef van Beveren, George Rupp, and Michael D. Scadron, Phys. Lett. **B495**, 300 (2000).
- [3] E. van Beveren and G. Rupp, Eur. Phys. J. **C10**, 468 (1999).
- [4] A.G.M. Verschuren, C. Dullemond and E. van Beveren, Phys. Rev. **D44**, 2803 (1991).
- [5] E. van Beveren, Nucl. Phys. **B21**, 43 (1991).
- [6] E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J. E. Ribeiro, Zeit. Phys. **C30**, 615 (1986).
- [7] E. van Beveren. Zeit. Phys. **C21**,291 (1984).
- [8] E. van Beveren, G. Rupp, T.A. Rijken, and C. Dullemond. Phys. Rev. **D27**,1527 (1983).
- [9] E. van Beveren, C. Dullemond, and G. Rupp. Phys. Rev. **D21**,772; (E) D22, 787 (1980).