RG vs PSS, cont'd

Thus, the PSS transf-n is a special case of RG one,

$$R_t \to S_t = \{x \to xt^{-1},\ g \to gt^{\nu}\}\ . \qquad (1)$$

Generally, in RG, instead of a power law, one has arbitrary functional dependence. Hence, one can consider all the RG transf-s as *functional* generalizations of PSS transf-n.

It is natural, to refer to them as to transf-s of funct'l scaling or **F**unctional **s**elf-**s**imilarity(FSS) transf-n.

In short

$$\mathrm{RG} \equiv \mathrm{FSS}$$

Other class example, Weak shock wave

Another example take from hydrodynamics. Turn to weak shock wave in the 1-dim case. The velocity-distance (from the creation point) plot at given moment has a triangular shape shown in Fig. 1

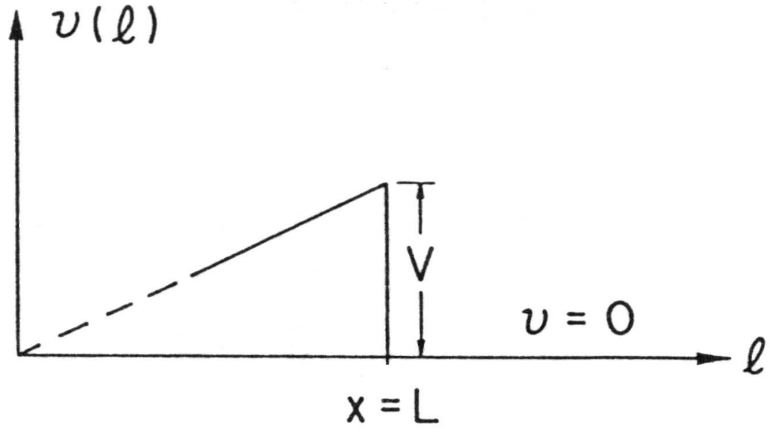

Figure 1: Weak shock wave shape.

and described by $v(l) = \frac{\ell}{L} V \, , \quad l \leq x, \quad = 0 \; \ell > \; x$. Here, $x = x(t)$ is the front position and $V = v(x)$ – the front velocity.

Weak shock wave

In the absence of viscosity the "consevation law" $LV = \mathrm{const}$ holds. Here, L and V coordinate and velocity at the shock wave front, are functions of time. However, they can be treated as functions of the front wave position $L = x, V = V(x)$. If the physical medium is homogeneous then the front velocity $V(x)$ should be considered as a function of two additional relevant arguments – its own value $V_0 = V(x_0)$ at some precedent point $(x_0 < x)$ and of x_0 coordinate.

RG vs PSS, cont'd

In can be written down in the form : $V(x) = G(x/x_0, V_0)$. If we pick up three points x_0, x_1 and x_2 (as in the Fig. 2 under homogeneity

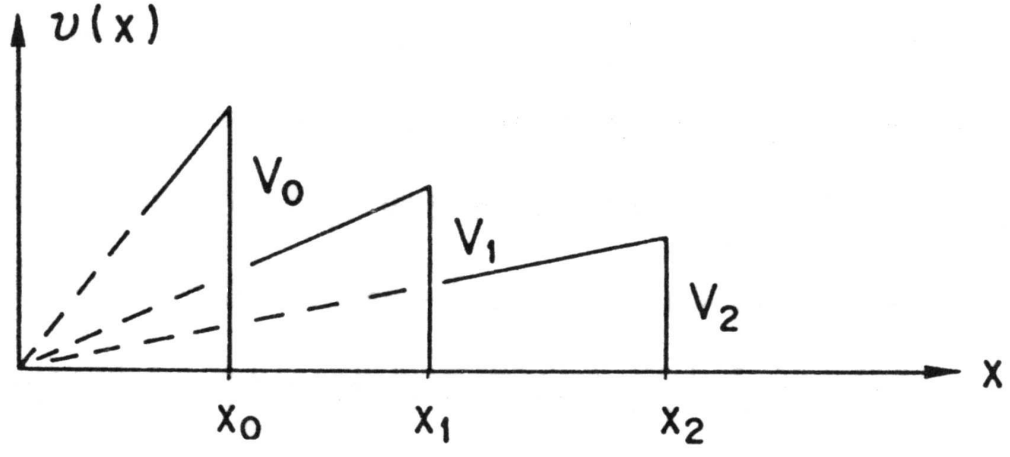

Figure 2: Weak shock wave evolution.

surrounding the initial condition may be given either at x_0 or x_1. Thus we obtain the FSS equation analogous to (2-11)

$$V_2 = G(x_2/x_0, V_0) = G(x_2/x_1, V_1) = G(x_2/x_1, G(x_1/x_0, V_0)) \ .$$

1-dim Transfer Problem

A similar argument is valid in 1-dimensional transfer problem. Take a half-space filled with a homogeneous medium, on surface of which some flow (of radiation or particles) with the intensity g_0 falls from the vacuum (see Fig. 3.

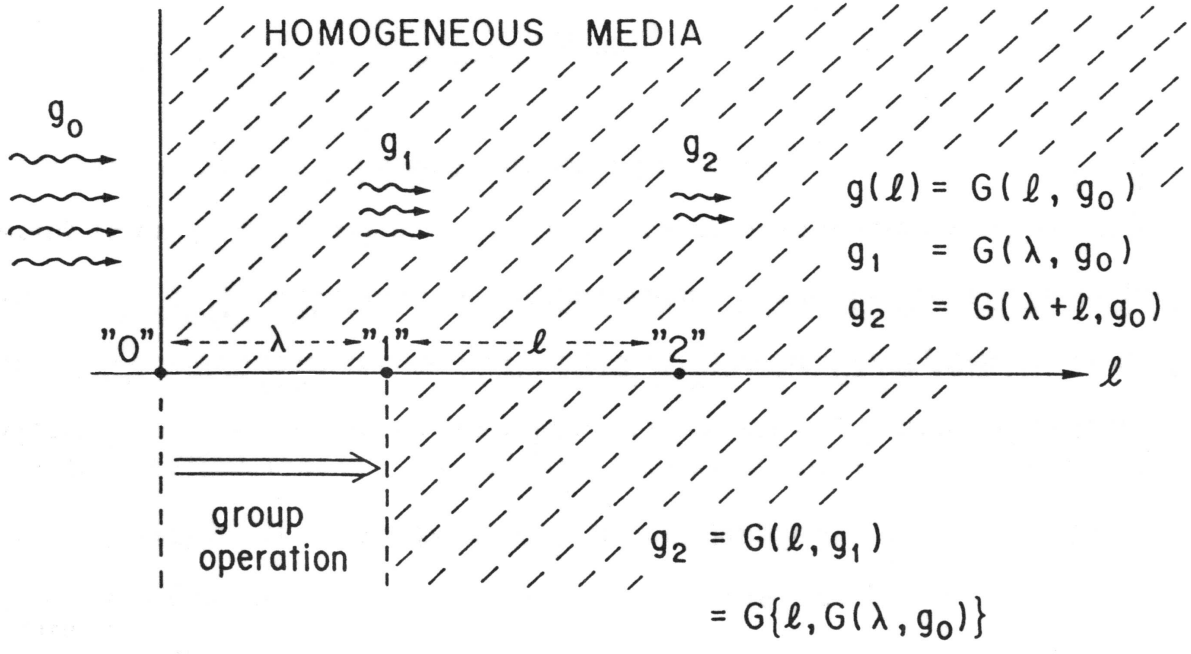

Figure 3: One dimensional transfer.

1-dim Transfer Problem, 2

Follow the flow as it moves inside medium at a distance l from boundary. Due to homogeneity along the l coordinate (distance from boundary), intensity of penetrated flow $g(l)$ is a function of 2 essential arguments, $g(l) = G(l, g_0)$.The flow values at points “O",“1", “2" shown in Fig. 3 can be related with each other by transitivity relations,

$$g_1 = G(\lambda, g_0) \ , \quad g_2 = G(\lambda+l, g_0) = G(l, g_1) \ , \ (2.11)$$

which lead to Eq. (7), i.e., the additive version of FSS FE.

1-dim Transfer Problem, 3

For discrete inhomogeneity, turn to the case of 2 diverse kinds of homogeneous materials separated by inner boundary surface at $l = L$ as in Fig. 5.

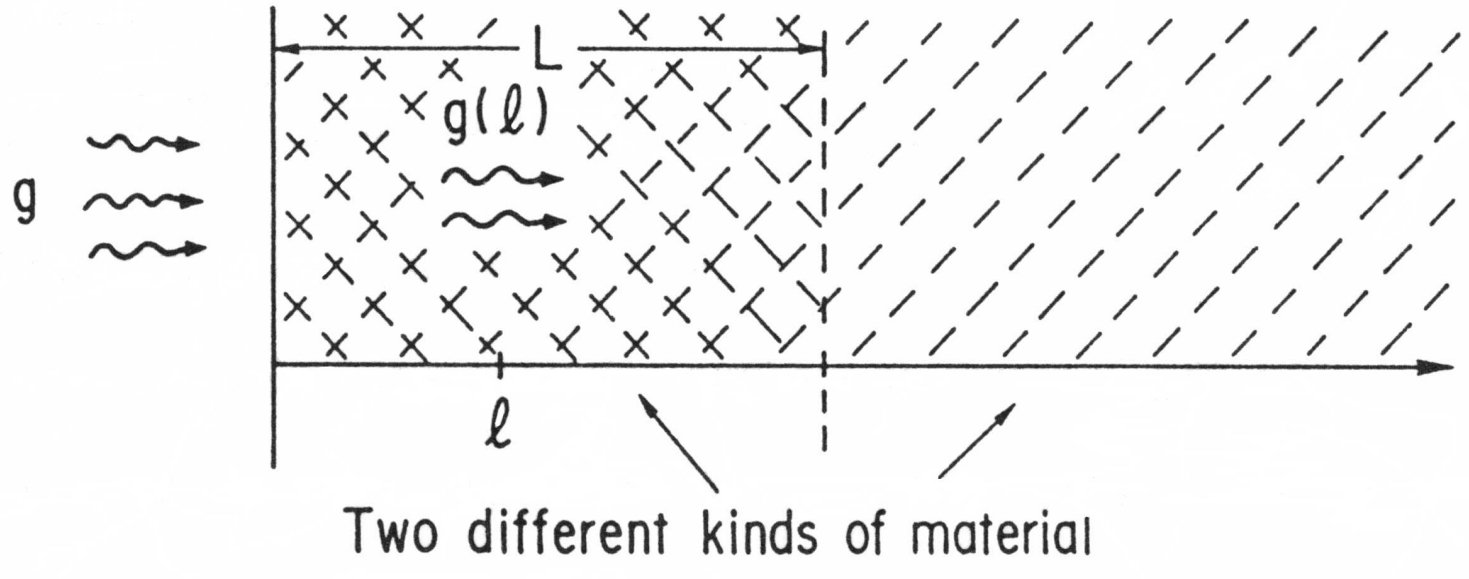

Figure 4: Transfer with inner boundary.

1-dim Transfer Problem, 4

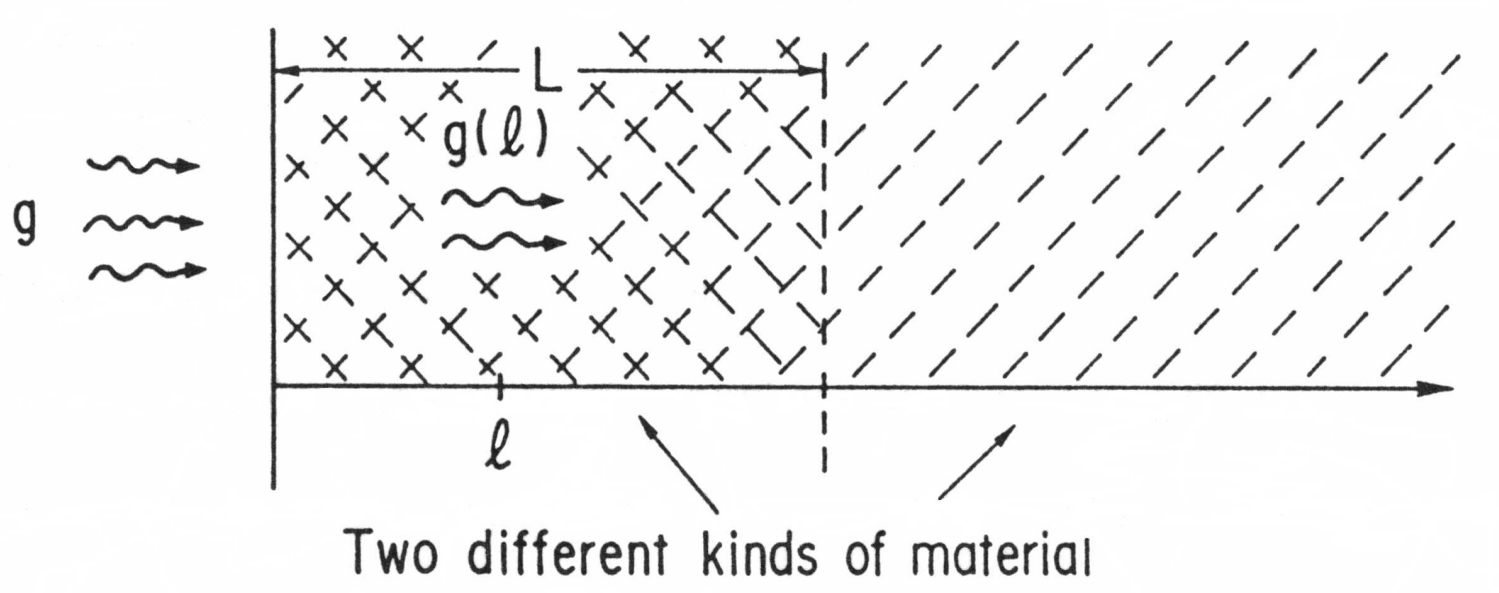

Figure 5: Transfer with inner boundary.

As in the elastic rod case, the point of homogeneity breaking $l = L$ may correspond to boundary with empty space, resulting FE coinciding with eq.(2.11).

1-dim Transfer Problem, 5

Transfer problem admits generalization related to “multiplication" of last argument as expressed by eq.(2.22) - in the case of radiation of 2 frequencies ω_1, ω_2 (or particles of different energies or of 2 various types) as shown in Fig. 6.

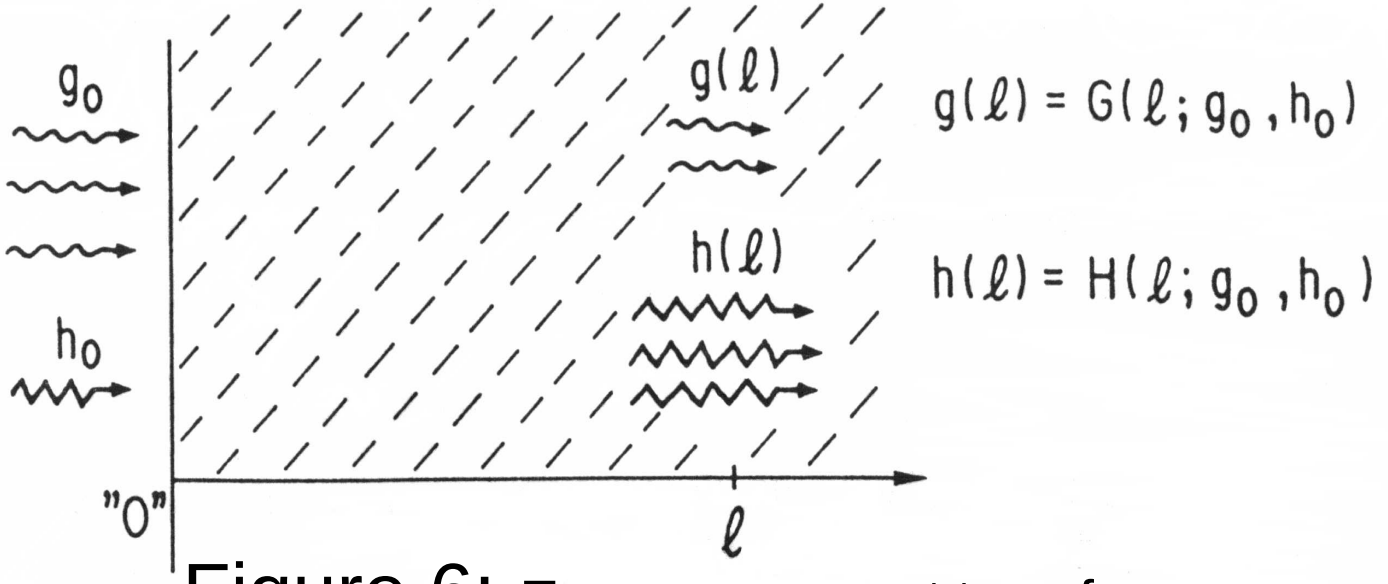

Figure 6: Two - component transfer

1-dim Transfer Problem, 6

Suppose that due to the medium material, the transfer processes of 2 flows are not independent. Then, the characteristic functions G and H are dependent on the both boundary values g_o, h_o and should be considered as 3-argument functions

$$g(l) = G(l, g_0, h_0) \ , \ h(l) = H(l, g_0, h_0) \ .$$

Group operation $l \to l - \lambda$ results in coupled set of FEqs:

$$G(l+\lambda, g, h) = G(l, g_\lambda, h_\lambda), \ H(l+\lambda, g, h) = H(l, g_\lambda, h_\lambda);$$

$$g_\lambda \equiv G(\lambda, g, h) \ , \ h_\lambda \equiv H(\lambda, g, h),$$

- additive version of system Eq. (2.22) at $k = 2$.

Nature of FSS=RG

Important conclusion is that a basic property yielding funct'l group eqs is just the transitivity property of a physical quantity with respect to the way of giving its boundary or initial value.

Hence, the RG=FSS symmetry is a symmetry of solution (or of eqs. and boundary conditions taken as a whole).

RG in QED

Effective Electron Charge. An essential feature of QFT is the presence of virtual states and of virtual transitions. In QED, process of virtual dissociation of a photon into an electron-positron pair and vice versa $\gamma \leftrightarrow e^+ + e^-$ can take place. The sequence of 2 such transitions can be drawn as a simple Feynman diagram presented in Fig. 7.

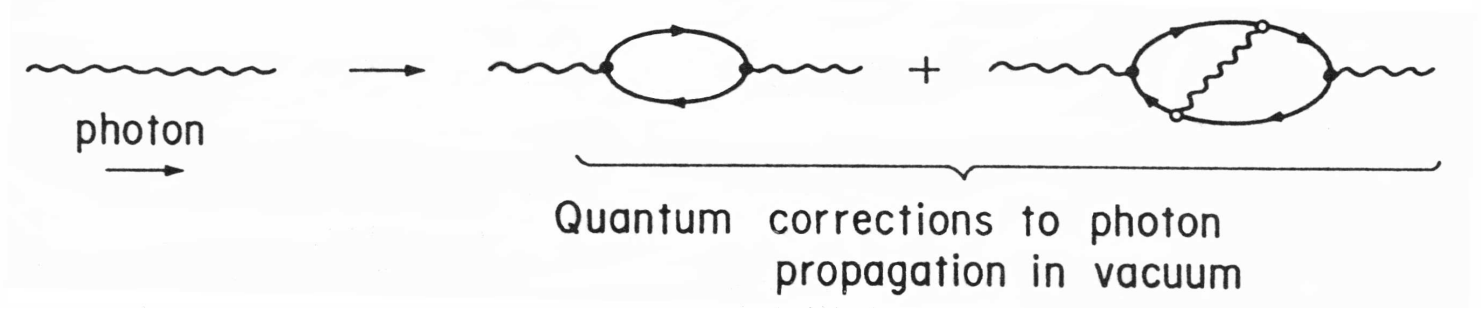

Figure 7: Vacuum polarization in QED.

RG in QED, 2

The vacuum polarization processes lead to the notion of *effective electron charge*. To explain, let's start with a classical analogy. Take polarizable medium consisting of molecules that form electric dipoles. Insert into it an external charge Q. Due to attraction of opposite charges, elementary dipoles change their position so that the charge Q will be partially screened. As a result, at distance r from Q potential will be smaller than vacuum Coulomb value Q/r taking form $Q(r)/r$ with $Q(r) \leq Q$. The quantity $Q(r)$ is known as an *effective charge*.

RG in QED, 3

As r decreases, $Q(r)$ increases and $Q(r \to 0) \to Q$. In QFT, the vacuum space itself stands for "polarizable medium". Quantum-field vacuum, physically, is not empty being filled with vacuum fluctuations, i.e., with virtual particles. These "zero fluctuation" are well-known property of ground state in quantum world. In QED, zero oscillations consist mainly of short-lived virtual (e^+, e^-) pairs that play role of tiny electric dipoles. The influence of these fluctuations we consider in detail.

RG in QED, 4

Fig.8 shows Feynman diagrams for process of measuring the electron charge. The electron under measurement is denoted by E. The measurement is performed by classic field $A_{\rm ext}$.

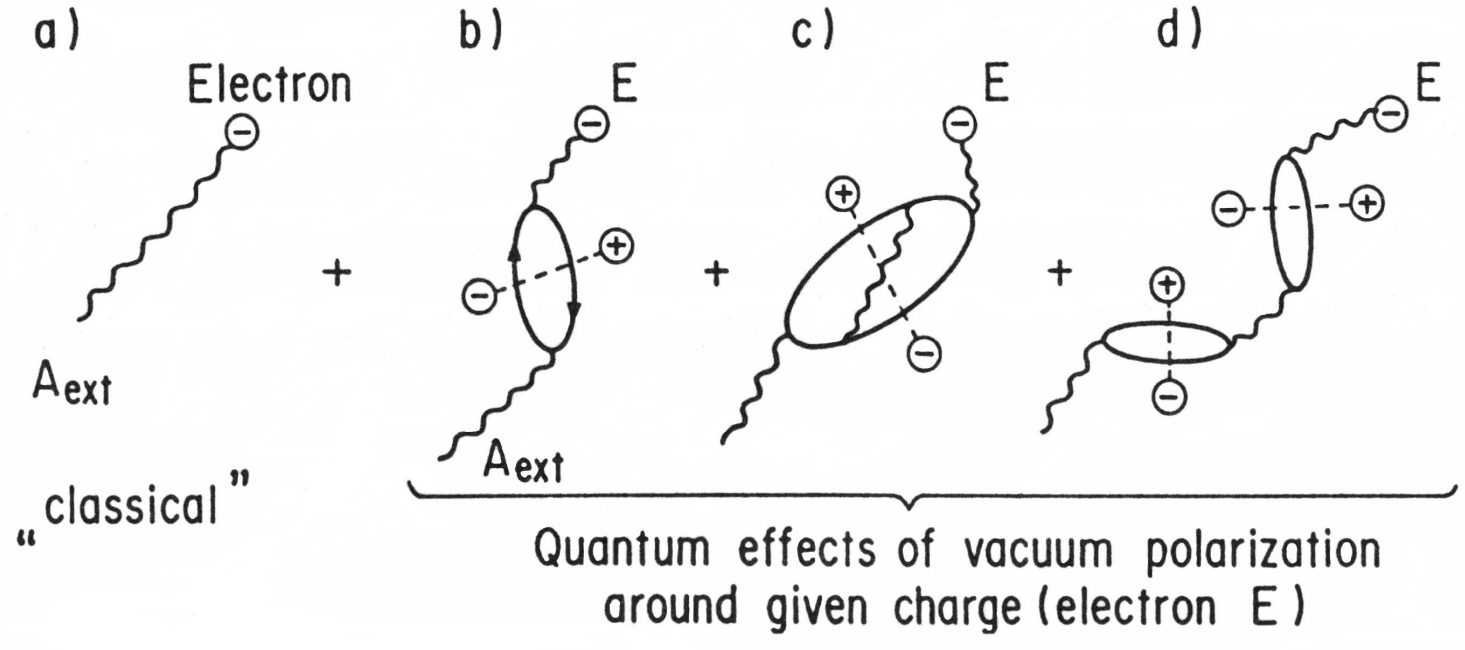

Figure 8: Electron charge measurement: a) classic, b) quantum .

RG in QED, 5

Fig.8a represents classical picture without quantum effects. In Fig.8b, the case with probing photon (quantum of ext. field) virtually dissociates into (e^+, e^-) pair. This pair forms a virtual dipole that yields screening of measured charge. As this process involves 2 electromagnetic acts, its contribution is proportional to small number $e^2 \equiv \alpha \simeq 1/137$; and this contribution depends on the distance r !

RG in QED, 6

In the region of r values much smaller than the electron Compton length $r_e = h/mc \sim\ 3,9.10^{-11} cm$, it depends on distance r logarithmically and is

$$e\ \rightarrow\ e(r)\ =\ e\left\{1 - \frac{\alpha}{3\pi}\ \ln\frac{r}{r_e}\ +\ O(\alpha^2)\right\}.$$

Here terms $\sim \alpha^2$ are related to processes shown on Figs. 8c,d, and so on.

RG in QED, 7

The electron charge dependence on distance has a purely quantum origin (first discussed by Dirac in mid30s). Qualitatively, the QED effective-charge behavior corresponds to classical screening. This dependence shown in Fig. 9 by a set of curves.

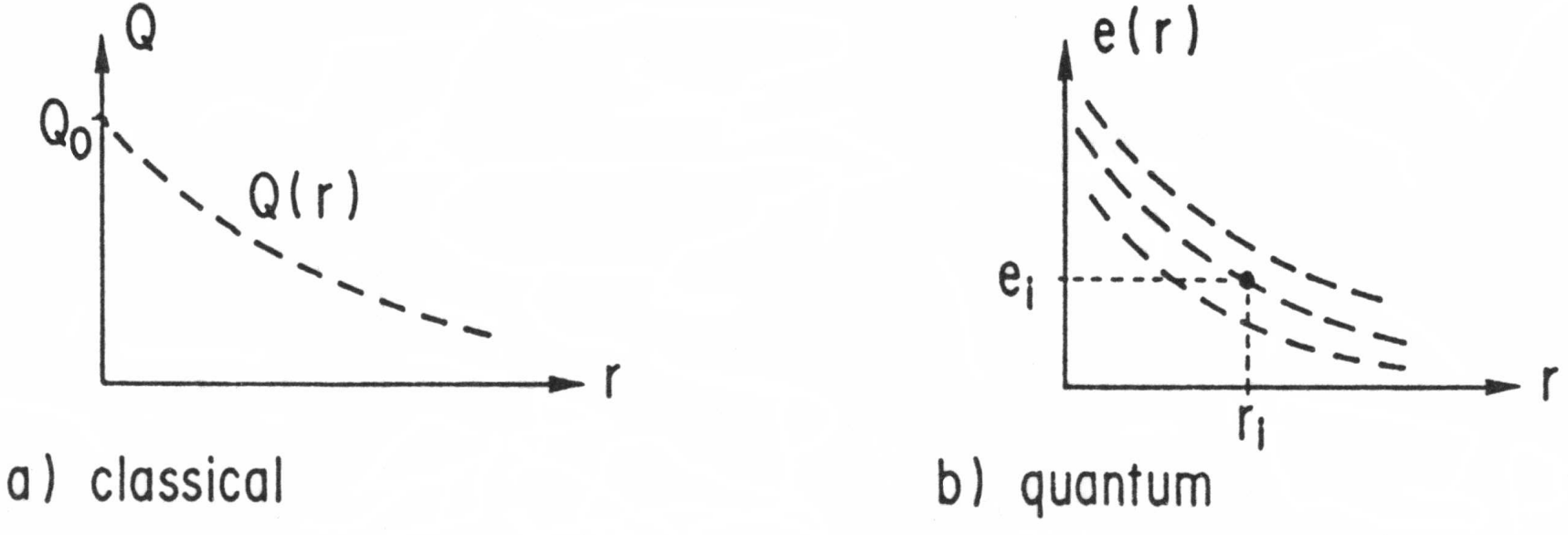

Figure 9: Theoretical possibilities for electron effective charge.

RG in QED, 8

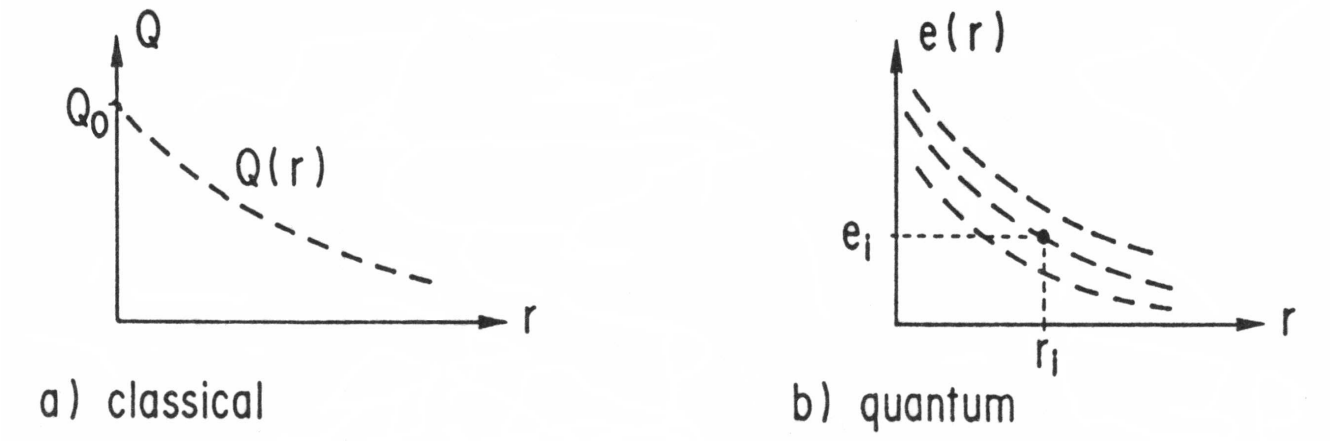

Figure 9: Theoretical possibilities for electron effective charge.

Each curve corresponds to possible behavior of effective charge $e(r)$ as obtained from theory and considered without any reference to experiment (parameter $e,\ \alpha = e^2$ being unspecified).

RG in QED, 9

In classical analogy, the value of external charge Q, inserted into polarizable medium, is known "from the very beginning" (by independent measurement). In quantum case it's not so and charge value can be measured only at *not very small* distances. The result of measurement can be specified by 2 quantities: the "distance of a measurement" r_i and the measured charge value e_i. Hence, to make choice from set of curves of Fig.(9) one has to fix point on plane with coordinates $r = r_i,\ e(r) = e_i$. Thus, for the chosen curve $e(r_i) = e_i$. Note, that the usual definition of an electron charge by macroscopic type (like Millikan) experiments corresponds to very large distances $r \gg r_e$, i.e., $1/137 = e^2(r = \infty)$.

RG in QED, 10

As it's well known, in microphysics one use momentum representation. Instead of $e(r)$, one deals with quantity $\bar{\alpha}(Q^2)$, a Fourier transform of $e(r)$ squared. It is monotonically increasing function of Q^2, 4-momentum transfer squared. The bar denotes a function (to distinct from $\alpha,\ \alpha_\mu,\ \alpha_i$ - its numerical values at some value of Q^2 argument). The correspondence condition with classical ED takes form $\bar{\alpha}(0) = 1/137$, as in HE scale usual classical field corresponds to a photon with zero 4-momentum.

However, as before, to fix one of possible curves on the plane $(Q,\ \bar{\alpha})$ one has to give the point $Q\ =\ \mu$, $\bar{\alpha} = \alpha_\mu$ and, hence, for the selected curve (shown in Fig.10) one has $\bar{\alpha}(\mu^2) = \alpha_\mu$.

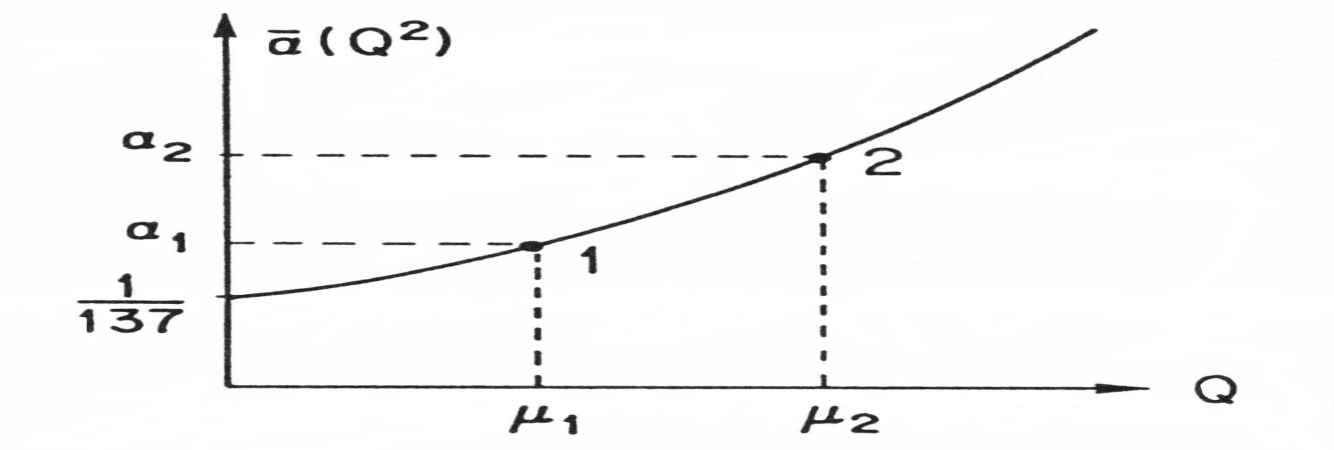

Figure 10: Various possibilities for parameterization ... .

RG in QED, 12

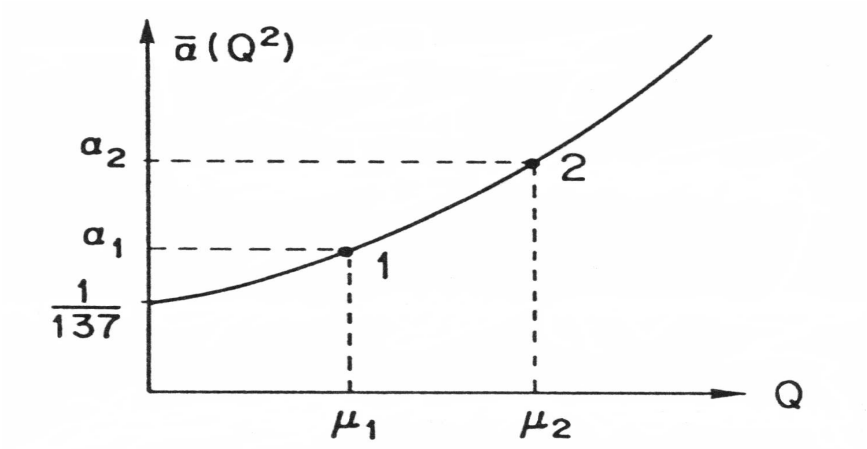

Figure 10: Various possibilities for parameterization ... .

Parameter μ often is referred to as a *scale parameter*. It's just momentum of photon used for a charge measurement. The effective coupling function $\bar{\alpha}(Q^2)$ describes the dependence of electron charge value on measurement conditions.

RG in QED, 13

The parameter μ has no analogue in classical ED Lagrangian. Its arising in QFT results is connected with term "dimensional transmutation". As was shown above, its appearance is very natural.

Recall ideas of mid30s by Niels Bohr related to the complementarity principle. The point is that to specify the quantum system, it is necessary to fix its ”macroscopical surrounding”, i.e., to give properties of macroscopic devices used in measurement. Just these devices are described by additional parameters.

RG in QED, 14

In our days the logarithmic corrections to the Millikan value are measured at big accelerators.

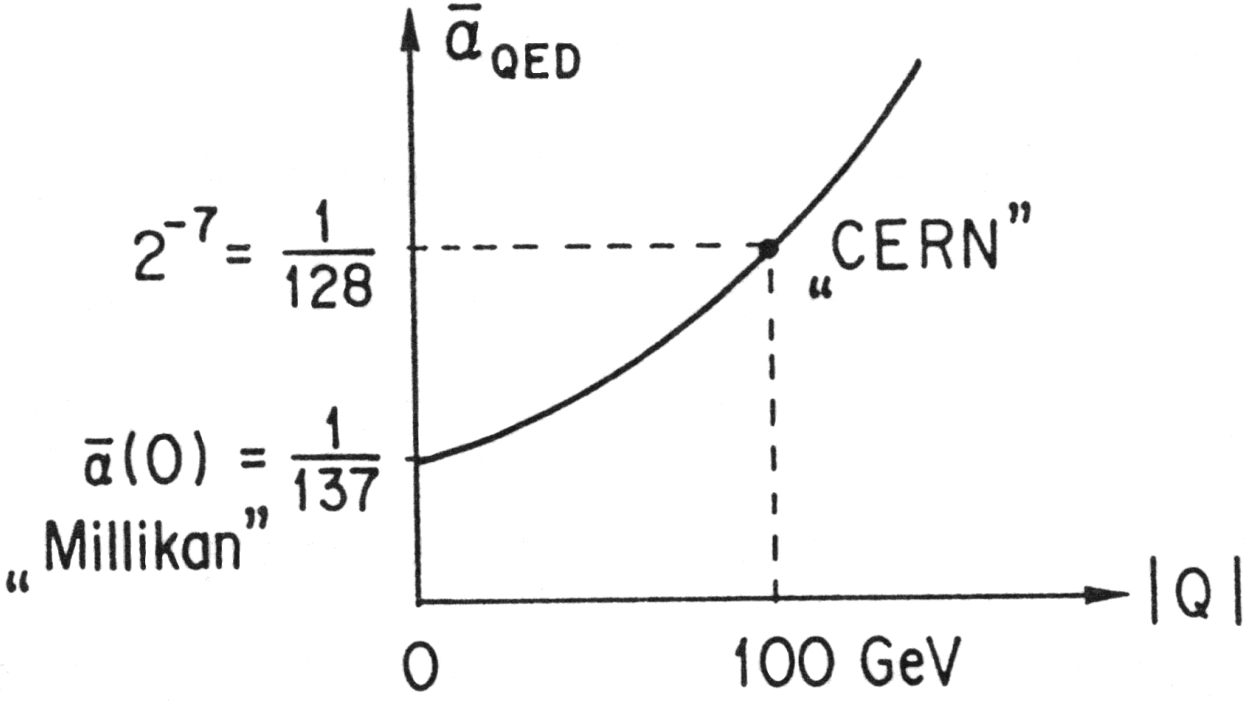

Figure 11: QED effective coupling vs data

Nature of FSS=RG

Important conclusion is that a basic property yielding funct'l group eqs is just the transitivity property of a physical quantity with respect to the way of giving its boundary or initial value.

Hence, the RG=FSS symmetry is a symmetry of solution (or of eqs. and boundary conditions taken as a whole).