Quark models in physics of strong interactions: from the MIT bag to chiral symmetry

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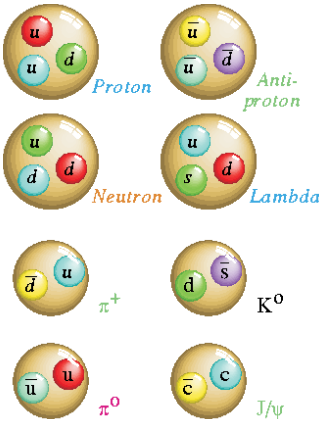
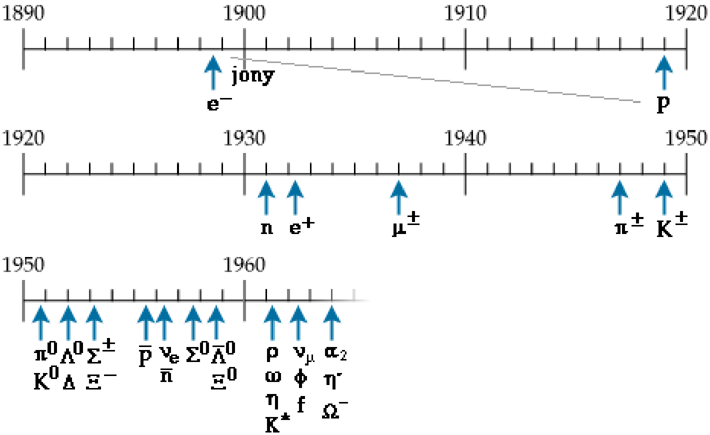
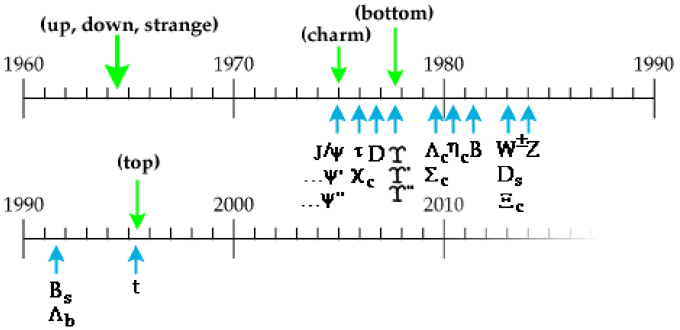


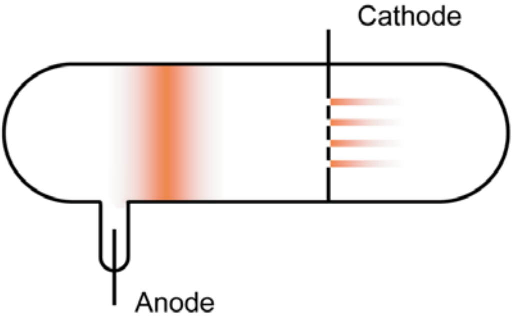
[James Joyce, Finnegan's Wake]

...
Three quarks for Muster Murk!
Sure he hasn't got much of a bark
...

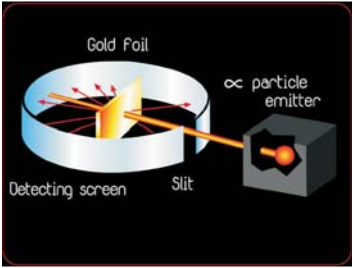

1890
1900
1910
1920
jony
e⁻
p
1920
1930
1940
1950
n
e⁺
μ±
π±
K±
1950
1960
π⁰ Λ⁰ Σ±
K⁰ Λ Ξ⁻
p̄ νe Σ⁰ Λ̄⁰
n̄ Ξ⁰
ρ νμ α₂
ω φ η'
η f Ω⁻
K*

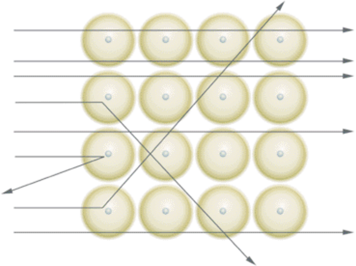

(up, down, strange)
(charm)
(bottom)
1960
1970
1980
1990
J/ψ
...ψ'
...ψ''
τ
D
χc
Υ
Υ'
Υ''
Λc
Σc
ηc
B
W±Z
Ds
Ξc
(top)
1990
2000
2010
Bs
Λb
t

- 1886 Eugen Goldstein, then Wilhelm Wien in 1898 followed by Joseph J. Thompson in 1910 observe positively charged particles of mass of the hydrogen atom in experiments with ionized gases in discharge tubes (canal rays)



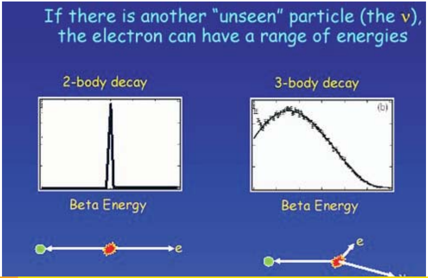
- 1909 Hans Geiger, Ernest Marsden i Ernest Rutherford scatter α particle on the fold foil and conclude existence of small, heavy, positively charged part



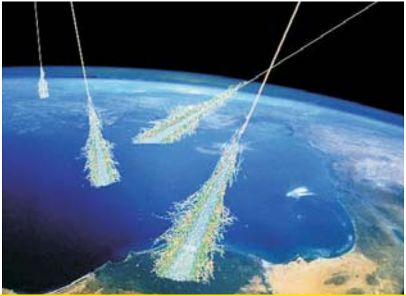


- 1911 Rutherford proposes the nucleus
- 1919 Rutherford shows that nitrogen bombarded by α particles emits positively charge particles of mass of hydrogen, in 1920 calls them *protons*

- 1921 James Chadwick and E. S. Bieler propose the concept of nuclear forces
- 1926 Erwin Schroedinger writes his equation
- 1928 Paul A. M. Dirac finds his equation for relativistic elementary particles of spin $1/2$
- 1930 Wolfgang Pauli postulates the existence of neutrino to explain the β decay



- 1932 James Chadwick discoveres the neutron bombarding beryl with α particles
- 1933 Enrico Fermi postulates the weak interactions
- 1933-35 Hideki Yukawa proposes the theory of strong interactions based of π-meson exchange
- 1936 Discovery of the muon (μ) in cosmic rays
- 1947 Discovery of $\pi^{\pm}$ in cosmic rays



- 1947 Richard Feynman invents his diagrams
- 1949 Discovery of K^+ via its decay - strange particles

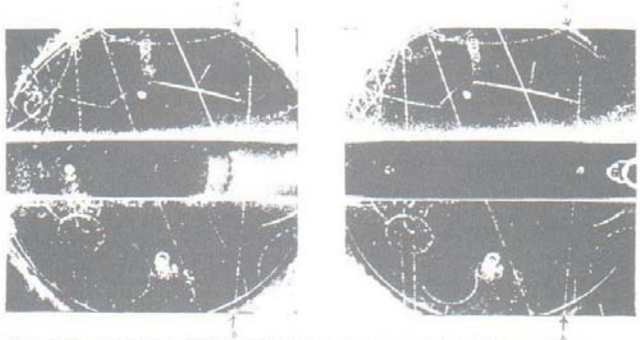
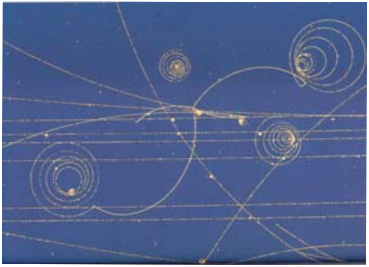


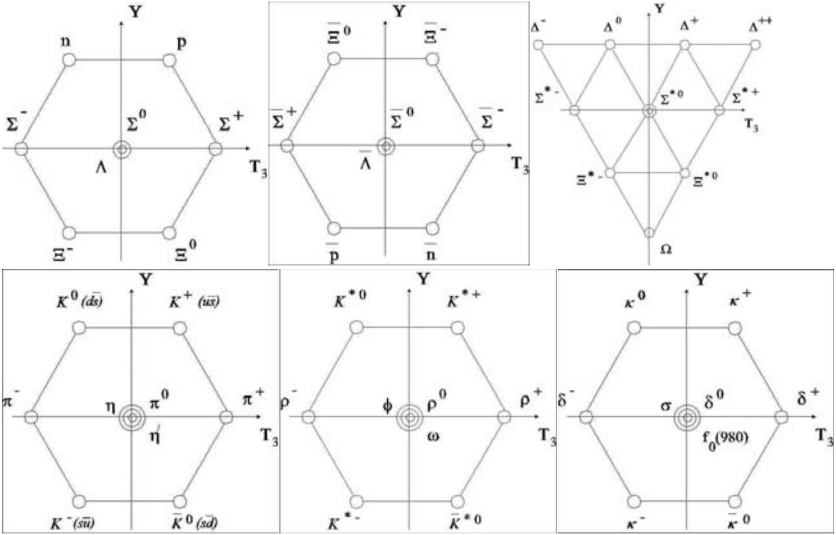
Fig. 1. STEREOSCOPIC PHOTOGRAPHS SHOWING AN UNUSUAL FORK (a b). THE DIRECTION OF THE MAGNETIC FIELD IS SUCH THAT A POSITIVE PARTICLE COMING DOWNWARDS IS DEVIATED IN A CLOCKWISE DIRECTION

- 1950 Discovery of π^0 in cosmic rays and in synchrocyclotron in Berkeley
- 1951 Discovery of Λ^0 i K^0 in cosmic rays

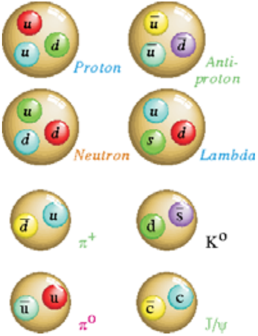
- 1952 Discovery of the Δ isobar with four charges: Δ^{++}, Δ^{+}, Δ^{0},Δ^{-}
- 1953– Avalanche of particle discoveries

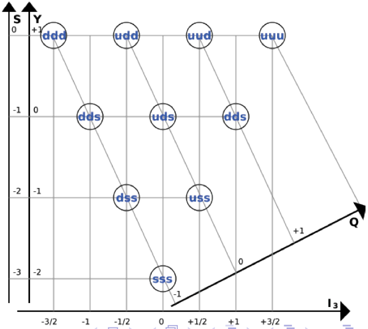


- 1953 Marian Danysz and Jerzy Pniewski discover hypernuclei
- 1953-57 Electron scattering displays the structure on nuclei
- 1954 Chen Ning Yang and Robert Mills invent the non-abelian gauge theory

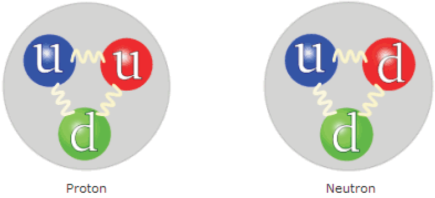

Y
n
p
Σ⁻
Σ⁰
Σ⁺
Λ
T₃
Ξ⁻
Ξ⁰
Y
Ξ̄⁰
Ξ̄⁻
Σ̄⁺
Σ̄⁰
Σ̄⁻
Λ̄
T₃
p̄
n̄
Y
Δ⁻
Δ⁰
Δ⁺
Δ⁺⁺
Σ*⁻
Σ*⁰
Σ*⁺
T₃
Ξ*⁻
Ξ*⁰
Ω
Y
K⁰ (ds̄)
K⁺ (us̄)
π⁻
η
π⁰
π⁺
η′
T₃
K⁻ (sū)
K̄⁰ (sd̄)
Y
K*⁰
K*⁺
ρ⁻
ϕ
ρ⁰
ρ⁺
ω
T₃
K*⁻
K̄*⁰
Y
κ⁰
κ⁺
δ⁻
σ
δ⁰
δ⁺
f₀(980)
T₃
κ⁻
κ̄⁰

- 1961 Murray Gell-Mann and Yuval Ne'eman propose independently a simple classification scheme based on the group $SU(3)$, the eightfold way. Prediction of triply-strange Ω^-, discovered in 1964
- 1964 Murray Gell-Mann and George Zweig introduce quarks (aces) of three flavors: down, up, and strange.





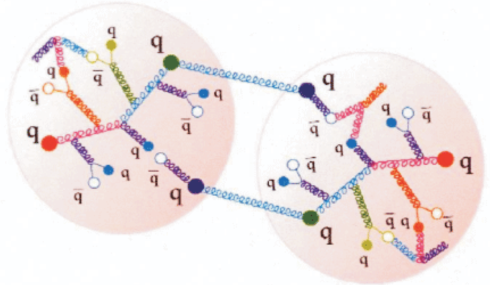
- 1964 Suggestion of existence of the fourth quark of a new flavor: charm, by Sheldon Glashow and James Bjorken
- 1965 Oscar W. Greenberg, M. Y. Han i Yoichiro Nambu introduce a new quantum number, color. It saves the spin-statistics theorem. All observed hadrons are “white”



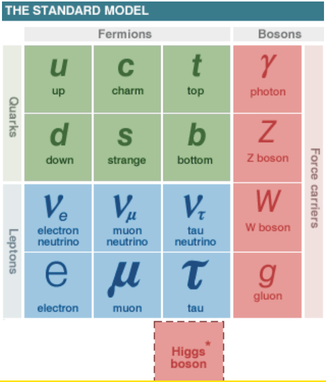
Quark composition of a proton and a neutron (diagrams from *Wikipedia*)

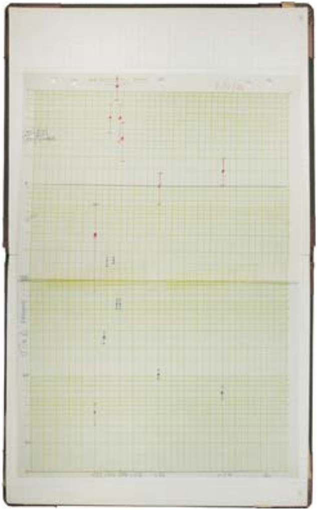
- 1967 Steven Weinberg and Abdus Salam unify the electromagnetic and weak interactions. Postulate of the Z_0 boson and the Higgs particle

- 1968-69 Discovery of partons in SLAC. James D. Bjorken and Richard Feynman explain the deeply inelastic scattering of electrons on protons by indicatind small, weakly-interacting *partons* inside the proton
- 1973 Harald Fritzsch and Murray Gell-Mann construct the *quantum chrodynamics (QCD)*, which is a gauge theory of interacting quarks and gluons



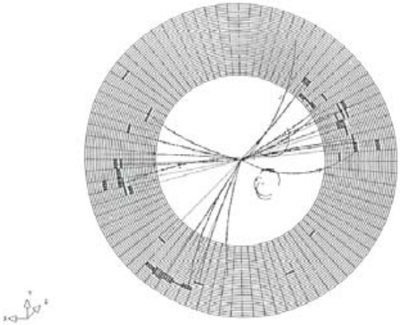
- 1973 David Politzer, David Gross and Frank Wilczek show that QCD is asymptotically free at short distances, thus partons may be identified with quarks
- 1974 Birth of the Standard Model of electroweak and strong interactions



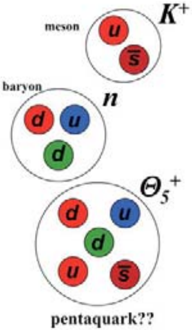


- 1974 Groups of Samuel Ting at BNL and Burton Richter at SLAC discover independently the J/ψ particle, which is a $c\bar{c}$ state
  "We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + Be \to e^+ + e^- + X$ by measuring the e^+e^- mass spectrum"
  "We have observed a very sharp peak in the cross section for $e^+e^- \to$ hadrons, e^+e^-, and possibly $\mu^+\mu^-$ at a center-of-mass energy of 3.105 ± 0.003 GeV. The upper limit to the full width at half-maximum is 1.3 MeV"

- 1976 Gerson Goldhaber and Francois Pierre discover the D^0 meson, $\bar{u}$ i c
- 1976 Martin Perl discovers the τ lepton, third generation
- 1977 Leon Lederman discovers the b quark in Fermilab
- 1979 Three-jet events in PETRA@DESY confirm existence of gluons



- 1983 Groups of Carlo Rubbia and Simon Van der Meer discover at CERN the $W^{\pm}$ and Z_0 bosons
- 1992 Discovery of neutrino oscillations in Kamiokande
- 1995 Eksperiments CDF and D0 at Fermilab discover the t quark
- 2003 "Discovery" of the pentaquark, $qqqq\bar{q}$



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Quarks

Once upon a time, there was a controversy in particle physics. There were some physicists [1] who denied the existence of structures more elementary than hadrons, and searched for a self-consistent interpretation wherein all hadron states, stable or resonant, were equally elementary. Others [2], appalled by the teeming democracy of hadrons, insisted on the existence of a small number of fundamental constituents and a simple underlying force law. In terms of these more fundamental things, hadron spectroscopy should be qualitatively described and essentially understood just as are atomic and nuclear physics [De Rújula, Giorgi, Glashow, 1975]

Quarks

	B	Q/e	I	I_3	s	c	b	t	m (MeV)	flavor
d	$1/3$	-1/3	$1/2$	-1/2	0	0	0	0	$3.5 \div 6$	down
u	$1/3$	$2/3$	$1/2$	$1/2$	0	0	0	0	$1.5 \div 3.3$	up
s	$1/3$	-1/3	0	0	-1	0	0	0	104^{+26}_{-34}	strange
c	$1/3$	$2/3$	0	0	0	1	0	0	1270^{+70}_{-110}	charmed
b	$1/3$	-1/3	0	0	0	0	-1	0	4200^{+170}_{-70}	bottom
t	$1/3$	$2/3$	0	0	0	0	0	1	171200 ± 2100	top

Gell-Mann – Nishijima formula:
$Q = e\left(I_3 + \frac{1}{2}(B + S + C + B + T)\right)$

Three families:

$$d, u, e, \nu_e$$
$$s, c, \mu, \nu_\mu$$
$$b, t, \tau, \nu_\tau$$

Analogy: positronium

e^- and e^+ with spins up or down: $|\, e^- \uparrow\rangle$, $|\, e^- \downarrow\rangle$, $|\, e^+ \uparrow\rangle$, $|\, e^+ \downarrow\rangle$
Denote $|\, e^+ \uparrow\rangle \otimes |\, e^- \uparrow\rangle = |\uparrow\uparrow\rangle$, etc. Metastable bound states form: para-positronium (singlet, antisymmetric, $C = (-)^{S+L} = 1$):

$$^1S_0, S = 0, S_z = 0 : \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

orto-positronium (triplet, symmetric, $C = (-)^{S+L} = -1$):

$$\begin{aligned} {}^3S_1, S = 1, S_z = 1 : &\quad |\uparrow\uparrow\rangle \\ S_z = 0 : &\quad \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ S_z = -1 : &\quad |\downarrow\downarrow\rangle \end{aligned}$$

Properties within the multiplet are the same, properties of different multiplets are different. **Group:** $SU(2)$

Since there are three light quarks, the relevant group is $SU(3)$, with generators of the algebra given by the Gell-Mann matrices:

$$\begin{aligned}
\lambda^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda^4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda^7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned}$$

The generators satisfy the $SU(3)$ algebra $[\lambda^b, \lambda^d] = i f_a^{.bd} \lambda^a$ with the structure constants $f_a^{.bd}$

Remarks:

The $SU(3)_F$ symmetry is not exact, as the masses of the light quarks u, d, s are not equal. Especially, $m_s \sim 150$ MeV is substantially larger than m_u and m_d. For that reason the members of multiplets acquire mass splitting. Additional mass splitting is due to Coulomb effects.

$SU(3)_F$ is global

$$3 \otimes \bar{3} = 8 \oplus 1$$

$J^P = 0^-$ mesons

K^0 (498) K^+ (494)

π^- (140) π^0 (135), η_8 π^+

K^- $\bar{K}^0$

η_0

meson octet

$d\bar{s}$ $u\bar{s}$

$d\bar{u}$ $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}, \frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ $u\bar{d}$

$s\bar{u}$ $s\bar{d}$

meson singlet

$$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

mixing of states:

$$\eta = \cos\theta_P \eta_8 - \sin\theta_P \eta_0$$
$$\eta' = \sin\theta_P \eta_8 + \sin\theta_P \eta_0$$
$$\theta_P = -10^o \div -20^o$$
$$m_\eta = 547\mathrm{MeV},\ m_{\eta'} = 958\mathrm{MeV}$$

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

baryon octet

udd uud

dds uds uus

dss uss

baryon singlet

uds

$J^P = \frac{1}{2}^+$ baryons

n (939.6) p (938.3)

Σ^- (1197.4) Σ^0 (1192.5) , Λ (1115.7) Σ^+ (1189.4)

Ξ^- (1321.3) Ξ^0 (1314.9)

$\Lambda(1405)$

baryon decuplet

ddd		udd		uud		uuu
	dds		uds		uus	
		dss		uss		
			sss			

$J^P = \frac{3}{2}^+$ baryons

Δ^- (1232)		Δ^0		Δ^+		Δ^{++}
	Σ^{*-} (1385)		Σ^{*0}		Σ^{*+}	
		Ξ^{*-} (1530)		Ξ^{*0}		
			Ω^- (1672)			

$J^P = 1^-$ mesons

K^{*0} (892) $\qquad K^{*+}$

ρ^- (770) $\qquad \rho^0\ ,\omega_8 \qquad \rho^+$

$K^{*-} \qquad \bar{K}^{*0}$

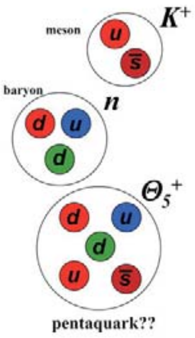
ω_0

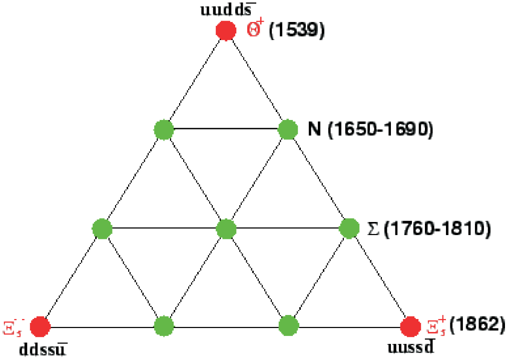
mixing of states:

$$\omega = \cos\theta_V \omega_8 - \sin\theta_V \omega_0$$

$$\phi = \sin\theta_V \omega_8 + \sin\theta_V \omega_0$$

$$\theta_V \simeq 35^o \simeq 1/\sqrt{2},\ m_\omega = 782\mathrm{MeV},\ m_\phi = 1020\mathrm{MeV}$$





Θ^+ should be very narrow, a few MeV

2003, LEPS@SPring-8 - "discovery" $\longrightarrow$ 600 papers $\longrightarrow$ 2007, CLAS@TJLAB - “undiscovery”

first report for the $N(\gamma, K^+K^-)X$ reaction

Color

$| \Delta^{++}, S_z = \frac{3}{2}\rangle = | u\uparrow \; u\uparrow \; u\uparrow\rangle$ – problem with the Pauli exclusion principle. Solution (Greenberg, Han, Nambu): a new quantum number, *color*, $SU(3)_c$

All states are neutral ("white"), *i.e.*, are singlets of color. Explicitly, the color wave function has the form

for mesons:

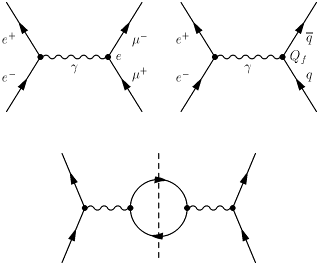
$$\frac{1}{\sqrt{3}} \, | \, q_a \bar{q}^a \rangle = \frac{1}{\sqrt{3}} \, | \, \left(r\bar{r} + b\bar{b} + g\bar{g} \right) \rangle$$

and for baryons:

$$\frac{1}{\sqrt{6}} \, | \, \epsilon^{abc} q_a q_b q_c \rangle = \frac{1}{\sqrt{6}} \, | \, (rbg + bgr + grb - brg - rgb - gbr) \rangle$$

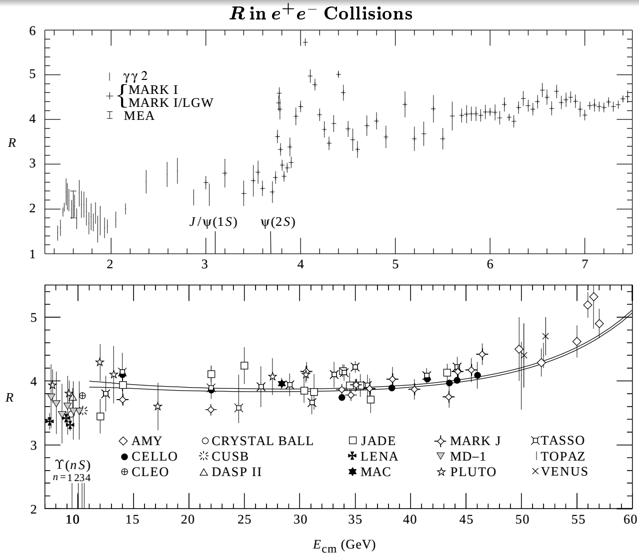
Evidence for color: R in e^+e^-

$$R = \frac{\sigma(e^+e^- \to \text{hadrony})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

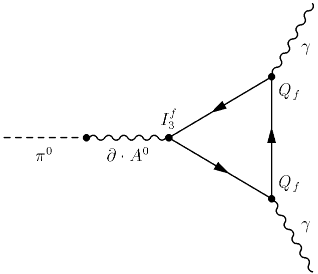


processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to q\bar{q}$ and squared amplitude $\sim N_c$

$R \simeq N_c \sum_{f=u,d,s,c,\ldots} Q_f^2 = 3\left(\frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \ldots\right)$
For energies above $2m_b$ we have $R \simeq \frac{33}{9}$


R in e^+e^- Collisions
γγ 2
MARK I
MARK I/LGW
MEA
$J/\psi(1S)$
$\psi(2S)$
R
E_{cm} (GeV)
AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
MD–1
PLUTO
TASSO
TOPAZ
VENUS
$\Upsilon(nS)$
$n=1\,2\,3\,4$

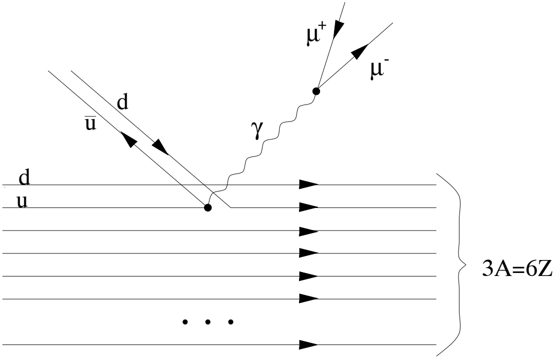
π^0 decay



Amplitude proportional to $N_c \sum_f I_3^f Q_f^2 = 3 \times 1/2 \times (4/9 - 1/9)$,
$\Gamma_{\pi^0 \to \gamma\gamma} = \frac{\alpha_{QED}^2 m_\pi^3}{64\pi^3 F_\pi^2}$
where $\alpha_{QED} = 1/137.04$ and $F_\pi = 93$ MeV is the pion decay constant.
The formula gives $\Gamma_{\pi^0 \to \gamma\gamma} = 7.6$ eV, while experimentally $\Gamma_{\pi^0 \to \gamma\gamma} = (7.4 \pm 1.5)$ eV. Without the color the theoretical result would be 9 times smaller!

Drell-Yan processes

high-energy scattering off symmetric nuclei ($N_u = N_d$)



$\frac{\sigma(\pi^- A \to \mu^+\mu^- X}{\sigma(\pi^+ A \to \mu^+\mu^- X} \simeq \frac{(2/3)^2}{(1/3)^2} = 4$ - agrees with exp.
(yet one more evidence for the quark charge assignement)

The quark-model wave functions have the generic form

$$\Psi_{j,m;I,I_3}(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \phi_c \Phi_{sf} \chi(\vec{x}_1 - \vec{x}_2, \vec{x}_2 - \vec{x}_3)$$

where ϕ_c is the color, Φ_{sf} the spin-flavor, and χ the spatial part. The coordinate of the ith quark is x_i. From translational invariance χ depend on relative coordinates only. Function $\Psi_{j,m;I,I_3}$ satisfies the conditions:

1. is antisymmetric (Fermi-Dirac statistics)
2. is color-singlet

$$\phi_c = \frac{1}{\sqrt{6}} \left(|rbg\rangle + |bgr\rangle + |grb\rangle - |brg\rangle - |rgb\rangle - |gbr\rangle \right) = |[r, b, g]\rangle$$

In the ground state χ is symmetric. Since ϕ_c is antisymmetric, Φ_{sf} must be symmetric.

Let us define the symmetrization operator $\{.\}$:

$$\{a,b,c\} = \frac{1}{\sqrt{6}}(abc+cab+bca+bac+acb+cba)$$

$$\{a,a,b\} = \frac{1}{\sqrt{3}}(aab+aba+baa)$$

$$[a,b,c] = \frac{1}{\sqrt{6}}(abc+cab+bca-bac-acb-cba)$$

We start with the highest weight state of the decuplet, $J=3/2$, $I=3/2$:

$$|\Delta^{++}, m=\frac{3}{2}\rangle_{sf} = |u\uparrow u\uparrow u\uparrow\rangle$$

We apply to both sides the spin lowering operator $J^- = J_1 - iJ_2$ with the property

$$J^-|j,m\rangle = \sqrt{j(j+1)-m(m-1)}|j,m-1\rangle$$

Analogously,

$$I^-|I,I_3\rangle = \sqrt{I(I+1)-I_3(I_3-1)}|I,I_3-1\rangle$$

$$J^-|\Delta^{++}, m = \frac{3}{2}\rangle = \sqrt{3}|\Delta^{++}, m = \frac{1}{2}\rangle.$$

Operators J^- i I^- may be expressed via quark operators:

$$J^- = \sum_{i=1}^{3} J_i^-, \qquad I^- = \sum_{i=1}^{3} I_i$$

Then

$$\begin{aligned} J^-|\Delta^{++}, m = \frac{3}{2}\rangle_{sf} &= \sum_{i=1}^{3} J_i^-|u\uparrow u\uparrow u\uparrow\rangle = \\ = \quad & |u\downarrow u\uparrow u\uparrow\rangle + |u\uparrow u\downarrow u\uparrow\rangle + |u\uparrow u\uparrow u\downarrow\rangle \\ = \quad & \sqrt{3}|\{u\downarrow, u\uparrow, u\uparrow\}\rangle \end{aligned}$$

Combining above equations gives

$$|\Delta^{++}, m = \frac{1}{2}\rangle_{sf} = |\{u\downarrow, u\uparrow, u\uparrow\}\rangle$$

$$I^-|\Delta^{++}, m = \frac{1}{2}\rangle_{sf} = \sqrt{3}|\Delta^+, m = \frac{1}{2}\rangle_{sf}.$$

$$I^-|\Delta^{++}, m = \frac{1}{2}\rangle_{sf} = \sum_{i=1}^{3} I_i^-|\{u\downarrow, u\uparrow, u\uparrow\}\rangle$$
$$= \quad |\{d\downarrow, u\uparrow, u\uparrow\}\rangle + \sqrt{2}|\{d\uparrow, u\downarrow, u\uparrow\}\rangle.$$

Comparing,

$$|\Delta^+, m = \frac{1}{2}\rangle_{sf} = \frac{1}{\sqrt{3}}|\{d\downarrow, u\uparrow, u\uparrow\}\rangle + \sqrt{\frac{2}{3}}|\{d\uparrow, u\downarrow, u\uparrow\}\rangle,$$

which is a combination of two orthogonal spin-flavor wave functions $|\{d\downarrow, u\uparrow, u\uparrow\}\rangle$ i $|\{d\uparrow, u\downarrow, u\uparrow\}\rangle$. We may form another combination orthogonal to $|\Delta^+, m = \frac{1}{2}\rangle_{sf}$, the proton with spin up:

$$|p, m = \frac{1}{2}\rangle_{sf} = \sqrt{\frac{2}{3}}|\{d\downarrow, u\uparrow, u\uparrow\}\rangle - \frac{1}{\sqrt{3}}|\{d\uparrow, u\downarrow, u\uparrow\}\rangle$$

The state satisfies ${}_{sf}\langle\Delta^+, m=\frac{1}{2}|p, m=\frac{1}{2}\rangle_{sf}=0$ and ${}_{sf}\langle p, m=\frac{1}{2}|p, m=\frac{1}{2}\rangle_{sf}=1$. The neutron is obtained by replacing u with d:

$$|n, m=\frac{1}{2}\rangle_{sf}=\sqrt{\frac{2}{3}}|\{u\downarrow, d\uparrow, d\uparrow\}\rangle-\frac{1}{\sqrt{3}}|\{u\uparrow, d\downarrow, d\uparrow\}\rangle$$

We may check that the electric charge is OK:

$$Q_p=\langle p, m=\frac{1}{2}|\hat{Q}|p, m=\frac{1}{2}\rangle=\frac{2}{3}\left(-\frac{1}{3}+\frac{2}{3}+\frac{2}{3}\right)+\frac{1}{3}\left(-\frac{1}{3}+\frac{2}{3}+\frac{2}{3}\right)=1$$

The magnetic-moment operator is $\hat{\mu} = \mu \sum_{i=1}^{3} Q(i)\sigma_z(i)$
Using $\langle \uparrow |\sigma_z| \uparrow\rangle = 1$, $\langle \downarrow |\sigma_z| \downarrow\rangle = -1$, $\langle \uparrow |\sigma_z| \downarrow\rangle = 0$, we can compute

$$\begin{aligned}
\mu_p &\equiv \langle p, m = \frac{1}{2}|\hat{\mu}|p, m = \frac{1}{2}\rangle = \\
&\left[\frac{2}{3}\left(\frac{1}{3}+\frac{2}{3}+\frac{2}{3}\right)+\frac{1}{3}\left(-\frac{1}{3}-\frac{2}{3}+\frac{2}{3}\right)\right]\mu = \mu, \\
\mu_n &\equiv \langle n, m = \frac{1}{2}|\hat{\mu}|n, m = \frac{1}{2}\rangle = \\
&\left[\frac{2}{3}\left(-\frac{2}{3}-\frac{1}{3}-\frac{1}{3}\right)+\frac{1}{3}\left(\frac{2}{3}+\frac{1}{3}-\frac{1}{3}\right)\right]\mu = -\frac{2}{3}\mu
\end{aligned}$$

Then

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3},$$

while in experiment $\mu_n/\mu_p = -1.913/2.793 = -0.68$.

Examples of sf wave functions for mesons:

$$
\begin{aligned}
|\pi^+ >_{\rm sf} &= \frac{1}{2}(|u\overline{d} > + |\overline{d}u >)(|\uparrow\downarrow> - |\downarrow\uparrow>) \\
|\rho^+, m = 0 >_{\rm sf} &= \frac{1}{2}(|u\overline{d} > - |\overline{d}u >)(|\uparrow\downarrow> + |\downarrow\uparrow>) \\
|a_1^+, m = 0 >_{\rm sf} &= \frac{1}{2}(|u\overline{d} > + |\overline{d}u >)(|\uparrow\downarrow> + |\downarrow\uparrow>)
\end{aligned}
$$

(no restriction from symmetry, as q and $\bar{q}$ are different particles)

The charge-conjugation symmetry C and the G-parity are defined to act on the u i d quarks as follows:

$$
G = Ce^{i\pi I_2} = Ci\tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},
$$

$$
C|q >= |\overline{q} >, \; C|\overline{q} >= |q >
$$

$$
G|u >= |\overline{d} >, \; G|\overline{d} >= -|u >, \; G|d >= -|\overline{u} >, \; G|\overline{u} >= |d >
$$

Then, for any charged state

$$G|\pi^a >= -|\pi^a >, \quad G|\rho^a >= |\rho^a >, \quad G|a_1^a >= -|a_1^a >$$

Since G-parity is conserved by strong interactions, ρ cannot decay in 3 pions, while a_1 cannot decay in 2 pions. Electroweak interactions violate this rule

Remarks:

G-parity is a generalization of C, working for the charged states as well, while C is defined for the neutral states only

Eigenstates of G-parity imply a symmetry of the flavor wave function, although q and $\bar{q}$ are different objects!

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The QCD Lagrangian

Quantum Chromodynamics is the field theory of quarks and gluons with the local (gauge) $SU(3)_c$ symmetry:

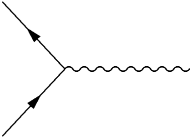
$$L_{QCD} = \sum_f \bar{\psi}_{\bar{c}} \left(D^\mu \gamma_\mu - m_f\right)_{\bar{c}c} \psi_c - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a, \quad c, \bar{c} = 1, 2, 3,$$

$$D^\mu = \partial^\mu - \frac{ig}{2} A^\mu_a \lambda^a,$$

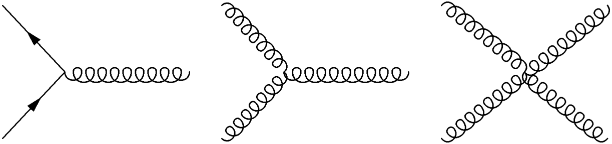
$$G^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^v A^\mu_a + \frac{ig}{4} f_a^{.bd} A^\mu_b A^\nu_d, \quad a, b, d = 1, ..., 8$$

where ψ_c is the quark field of color c and A^μ_a is the gluon field of Lorentz index μ and color a. Gluons are in an octet (adjoint) representation, hence there are $N_c^2 - 1 = 8$ of them. The symbols $f_a^{.bd}$ are the structure contants of $SU(3)$

Interaction vertices



QED



QCD

Gluons interact also among themselves (have charge). In QED photons do not interact directly, as they are neutral.

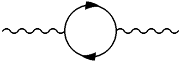
The gluon was indirectly observed in Petra@DESY in 1979. Three-jet events originate from an initial state containg q, $\bar{q}$, and a gluon

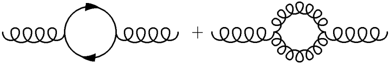
QED vs. QCD

QED	QCD
abelian gauge, $U(1)$ one kind of charge photons neutral	nonabelian gauge, $SU(3)_c$ 3 kinds of fundamental charge charged gluons
coupling constant grows with momentum transfer	coupling constant decreases with momentum transfer
screening	antiscreening
	asymptotic freedom color confinement
perturbative vacuum	nonperturbative vacuum condensates
accurate dynamical predictions	approximate dynamical predictions

Perturbative evolution of the coupling constant

$\alpha = \frac{g^2}{4\pi}$, the coupling constant "runs" with the scale





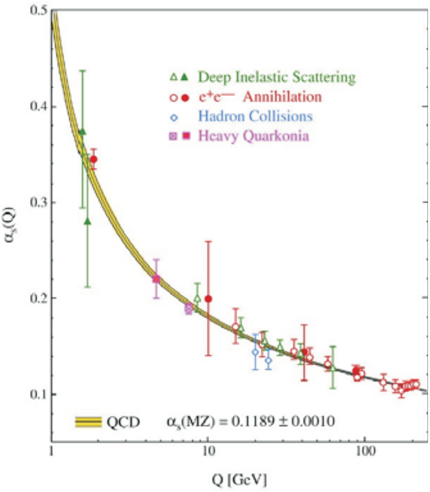
$$\alpha_{\rm QED}(Q^2) = \frac{\alpha_{\rm QED}(\mu^2)}{1 - \alpha_{\rm QED}(\mu^2)\frac{1}{3\pi}\log\frac{Q^2}{\mu^2}}$$

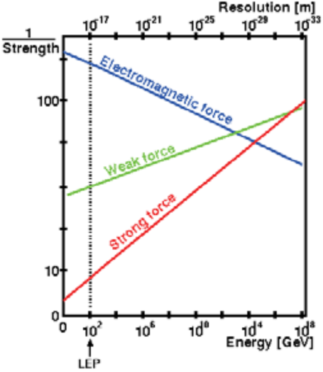
$$\alpha_{\rm QED}(m_e^2) = 1/137.035999679(94)$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{11N_c - 2N_f}{12\pi}\log\frac{Q^2}{\mu^2}}$$

$\alpha_{QED}(Q^2)$ increases with growing Q^2, or decreasing distance

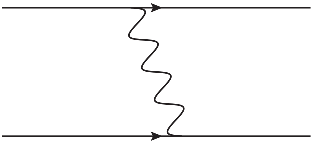
$\alpha_s(Q^2)$ decreases with growing Q^2, or decreasing distance when $11N_c > 2N_f$, true up to 5 flavors $\rightarrow$ asymptotic freedom

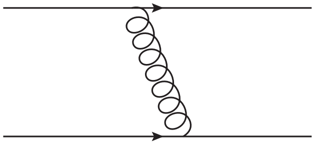

Deep Inelastic Scattering
e+e— Annihilation
Hadron Collisions
Heavy Quarkonia
QCD
αs(MZ) = 0.1189 ± 0.0010
αs(Q)
Q [GeV]


Resolution [m]
1/Strength
Electromagnetic force
Weak force
Strong force
Energy [GeV]
LEP

- no analytic proof of confinement in QCD
- one of the seven Millenium Problems posed by the Clay Mathematics Institute (1 million $ prize!)
- lattice gauge QCD calculations
- spectroscopy, no free color ever observed, coupling constant grows with distance

One-gluon-exchange potential





In QED the Coulomb interaction is $V_{\text{coul}}(r) = Q_1 Q_2 / r$. In QCD the charge is non-abelian and the qq interaction is proportional to the matrix element

$$\langle \phi_c | \sum_{a=1}^{8} \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2} | \phi_c \rangle.$$

Similarly, for $q\bar{q}$

$$\langle \phi_c | \sum_{a=1}^{8} \frac{\lambda_i^a}{2} \frac{-(\lambda_j^a)^*}{2} | \phi_c \rangle,$$

where $|\phi_c\rangle$ is the color wave function

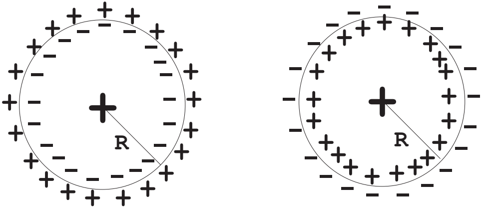
For the singlet

$$\begin{aligned} \langle \text{singlet } qqq | \sum_{a=1}^{8} \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2} | \text{singlet } qqq \rangle &= -\frac{2}{3}, \\ \langle \text{singlet } \bar{q}q | \sum_{a=1}^{8} \frac{\lambda_i^a}{2} \frac{-(\lambda_j^a)^*}{2} | \text{singlet } \bar{q}q \rangle &= -\frac{4}{3}, \end{aligned}$$

thus we have attraction. Furthermore,

$$\begin{aligned} \langle \text{sextet } qq | \sum_{a=1}^{8} \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2} | \text{sextet } qq \rangle &= \frac{1}{3}, \\ \langle \text{octet } \bar{q}q | \sum_{a=1}^{8} \frac{\lambda_i^a}{2} \frac{-(\lambda_j^a)^*}{2} | \text{octet } \bar{q}q \rangle &= \frac{1}{6}, \end{aligned}$$

which means repulsion.

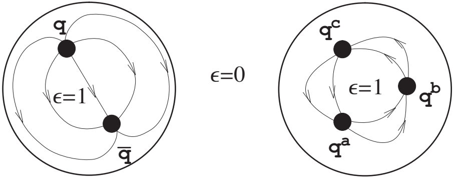


T. D. Lee: the QCD vacuum is a perfect dia-electric, *i.e.*, the dielectric constant $\varepsilon = 0$. Assume a localized color charge Q. according to the Gauss law, away fro the charge the electric induction field is $D = \frac{Q}{r^2}$. Furthermore, $D = \varepsilon E$, where E is the electric field. The energy density is then

$$\rho = ED = \frac{Q^2}{\varepsilon r^4}.$$

Evidently, when $\varepsilon = 0$, then $\rho = \infty$. Thus the isolated charge cannot exist in a perfect dia-magnetic.

Only singlet (charge-neutral) states may have finite energy. Quarks or gluons (coupled to a color singlet) may locally deform the vacuum, where $\varepsilon > 0$, forming bags. Outside of the bag $\varepsilon = 0$, while inside $\varepsilon = 1$.



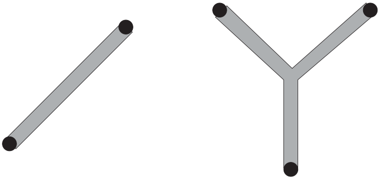
The D-field lines are completely contained inside the bag and are tangent to the walls, such that the perpendicular component of D is continuous across the wall. At the wall the appropriate charge density is induced, providing the desired field configuration.

Analogy between confinement in QCD and superconductivity in QED

QED		QCD
Meissner effect	$\leftrightarrow$	confinement
H	$\leftrightarrow$	E
$\mu_{\text{inside}} = 0$	$\leftrightarrow$	$\varepsilon_{\text{vac}} = 0$
$\mu_{\text{vac}} = 1$	$\leftrightarrow$	$\varepsilon_{\text{inside}} = 1$
inside	$\leftrightarrow$	outside
outside	$\leftrightarrow$	inside

Strings

The generation of the bag requires a certain work, thus the bags want to have a small volume (see the following part). When two color charges are separated, a string (flux-tube) is formed in between. At large separations d the potential energy is $V = a + \sigma d$, where a and σ (the string tension) are constants. Thus a constant force is exerted on the charge in the direction of the string. Phenomenologically, $\sigma \simeq 1\mathrm{GeV}/\mathrm{fm}$.

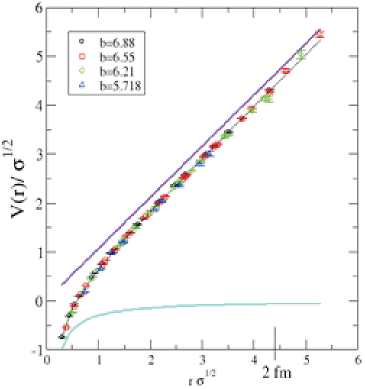


The Cornell potential:

$$V_{c\bar{c}} = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + f(r)\vec{S}_c \cdot \vec{S}_{\bar{c}}$$

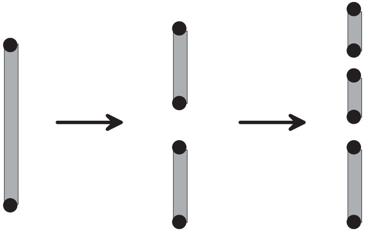
Lattice QCD simulations

The potential between two (infinitely heavy) charges



String breaking

At even larger separations it is favorable for the system to break into two strings (the Schwinger mechanism). This is used to model the hadron decays.



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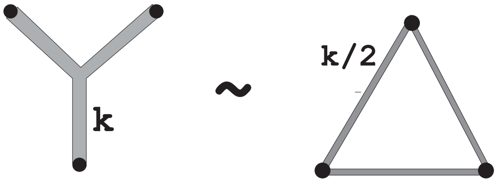
Nonrelativistic quark models

$\Delta = Y$ rule – replace the three-body force with two-body forces



Color charges: $F_i^a = \frac{1}{2}\lambda_i^a$ or $F_i^a = \frac{1}{2}(-\lambda_i^a)^*$

$$H = H_0 + H_{\rm conf} + \sum_{i<j} \left(H_{\rm coul}^{ij} + H_{\rm hyp}^{ij} + H_{\rm so}^{ij} \right) ,$$

$$H_0 = \sum_i \left(M_i + \frac{p_i^2}{2M_i} \right) , \quad H_{\rm conf} = \sum_{i<j} (br_{ij} + c) , \quad H_{\rm coul}^{ij} = \frac{\alpha_s}{r_{ij}} \sum_{a=1}^{8} F_i^a F_j^a$$

In analogy to atomic physics, large-M expansion of the one-gluon exchange [Karl and Isgur, 1977]

$$H_{\rm hyp}^{ij} = -\frac{\alpha_s}{M_i M_j}\left(\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(r_{ij}) + \frac{1}{r_{ij}^3}\left[\frac{3\vec{S}_i\cdot\vec{r}_{ij}\;\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^2} - \vec{S}_i\cdot\vec{S}_j\right]\right)$$

$$\times \quad \sum_{a=1}^{8} F_i^a F_j^a,$$

$$H_{\rm so}^{ij} = -\frac{\alpha_s}{r_{ij}^3}\left(\frac{1}{M_i}+\frac{1}{M_j}\right)\left(\frac{\vec{S}_i}{M_i}+\frac{\vec{S}_j}{M_j}\right)\cdot\vec{L}_{ij}\;\sum_{a=1}^{8} F_i^a F_j^a$$

$$- \quad \frac{1}{2r_{ij}}\frac{dH_{coul}^{ij}}{dr_{ij}}\left(\frac{\vec{S}_i}{M_i^2}+\frac{\vec{S}_j}{M_j^2}\right)\cdot\vec{L}_{ij},$$

The spin-orbit part leads to problems! – its first term usually dropped, inconsistency

Separation of the center-of-mass is possible. For baryons (3 quarks)

$$\begin{aligned} R &= \frac{M_1\vec{r}_1 + M_2\vec{r}_2 + M_3\vec{r}_3}{M_1 + M_2 + M_3}, \\ \vec{\rho} &= \frac{1}{\sqrt{2}}\left(\vec{r}_1 - \vec{r}_2\right), \quad \vec{\lambda} = \frac{1}{\sqrt{6}}\left(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3\right) \end{aligned}$$

The corresponding momenta are $\vec{P}$, $\vec{p}_\rho$ and $\vec{p}_\lambda$. The Hamiltonian is approximated with that of the harmonic oscillator. For the equal-mass case

$$H = M + \frac{\vec{P}^2}{2(3M)} + \frac{\vec{p}_\rho^2}{2M} + \frac{3}{2}k\rho^2 + \frac{\vec{p}_\lambda^2}{2M} + \frac{3}{2}k\lambda^2 \qquad (1)$$

Introduce $\beta = (3kM)^{1/4}$ and $\omega = (3k/M)^{1/2}$

The solution is

$$\Psi^{N,\sigma}_{LL_3} = \psi^{N,\sigma}_{LL_3} \frac{\beta^3}{\pi^{3/2}} e^{-\frac{1}{2}\beta^2(\rho^2+\lambda^2)},$$

with

$$\begin{aligned}
\psi^{0,S}_{00} &= 1, \\
\psi^{1,\rho}_{11} &= \beta\rho_+, \quad \psi^{1,\rho}_{10} = \beta\rho_0, \quad \psi^{1,\rho}_{1-1} = \beta\rho_-, \\
\psi^{1,\lambda}_{11} &= \beta\lambda_+, \quad \psi^{1,\lambda}_{10} = \beta\lambda_0, \quad \psi^{1,\lambda}_{1-1} = \beta\lambda_-, \\
\psi^{2,S}_{00} &= \frac{1}{\sqrt{3}}\beta^2(\rho^2+\lambda^2-\frac{3}{\beta^2}), \\
\psi^{2,\rho}_{00} &= \frac{2}{\sqrt{3}}\beta^2(\vec{\rho}\cdot\vec{\lambda}), \; \psi^{2,\lambda}_{00} = \frac{1}{\sqrt{3}}\beta^2(\rho^2-\lambda^2)...
\end{aligned}$$

Indices have the following meaning: N is the number of excited quanta of the harmonic oscillator, L and L_3 are the orbital angular momentum and its third component, σ denotes the symmetry of the wave function: S – symmetric, A – antisymmetric, ρ or λ – mixed. The symmetry of the spin-isospin wave function "compensates" such that the full space-spin-isospin wave function is symmetric

The energy of the ground state $(N+3)\omega$. The states group in multiplets of $SU(6)$, where $6 = 3 \text{ flavors } \times 2 \text{ spins}$. The notation is $[\dim, L_N^P]$ For the ground state we have $[56, 0_0^+]$, built on the wave function $\psi_{00}^{0,S}$. In contains to (spin degeneracy) flavor octets and four flavor decuplets. The firs excited state is $[70, 1_1^-]$, built on $\psi_{1M}^{1,\rho}$ and $\psi_{1M}^{1,\lambda}$, and so on. The anharmonic corrections generate the splitting pattern:

$$
\begin{aligned}
E[56, 0_0^+] &= E_0, \\
E[70, 1_1^-] &= E_0 + \Omega, \\
E[56', 0_2^+] &= E_0 + 2\Omega - \Delta, \\
E[70, 0_2^+] &= E_0 + 2\Omega - \frac{1}{2}\Delta, \ldots
\end{aligned}
$$

One also includes the strange quark mass effects.
Problems: impossibility to describe the pion, "missing" states, incorrect sequence of states (*e.g.* the Roper resonance), the ad hoc removal of the spin-orbit force
Successes: heavy quarkonia, separation of the center of mass, simplicity

Sample NRQM results

[S. Capstick, N. Isgur, 1986]

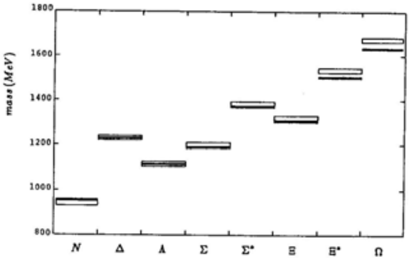


Figure 3: The Ground State Baryons.

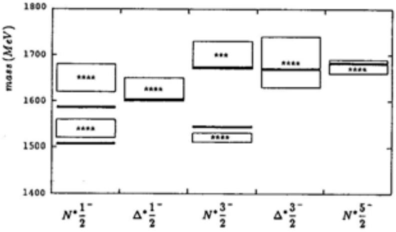


Figure 4: The Negative Parity S=0 Excited Baryons of the N=1 band: boxes show the experimental regions in which the resonances lie, bars show the predictions of the model for states that should be coupled, with ΔM=+50 MeV.

Exotics

The quark model provides the following quantum numbers for the $q\overline{q}'$ states:

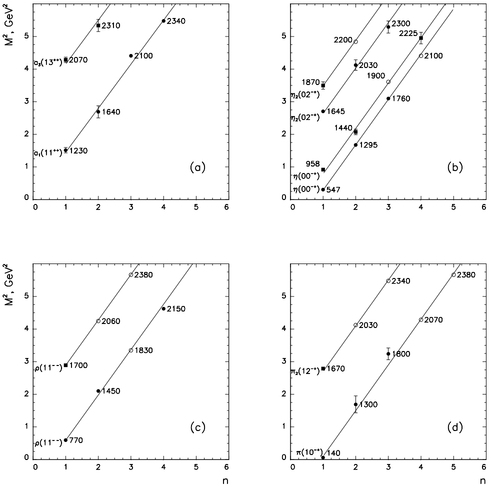
$$: \qquad J^{PC} = \begin{cases} 0^{-+}, 1^{+-}, 2^{-+}, 3^{+-}, 4^{-+}, 5^{+-}, \ldots & (S=0) \\ 0^{++}, 1^{--}, 1^{++}, 2^{--}, 2^{++}, 3^{--}, 3^{++}, \ldots & (S=1) \end{cases}$$

Thus $P=(-1)^{L+1}$, $CP=(-1)^{S+1}$, moreover $G=C(-1)^I$. Other combinations of quantum numbers are exotic

$$\text{exotic states:} \qquad J^{PC} = 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$$

They may be *glueballs, hybrids, eg.* $q\overline{q}g$*, mesonic molecules (states* $\overline{q}\overline{q}qq$*)*. So far there is no convincing evidence for such states. Note that glueballs, hybrids, etc. may exist with nonexotic quantum numbers as well.

Regge trajectories



J - spin, n - radial excitation

$$m^2 = m_0^2 + aJ$$

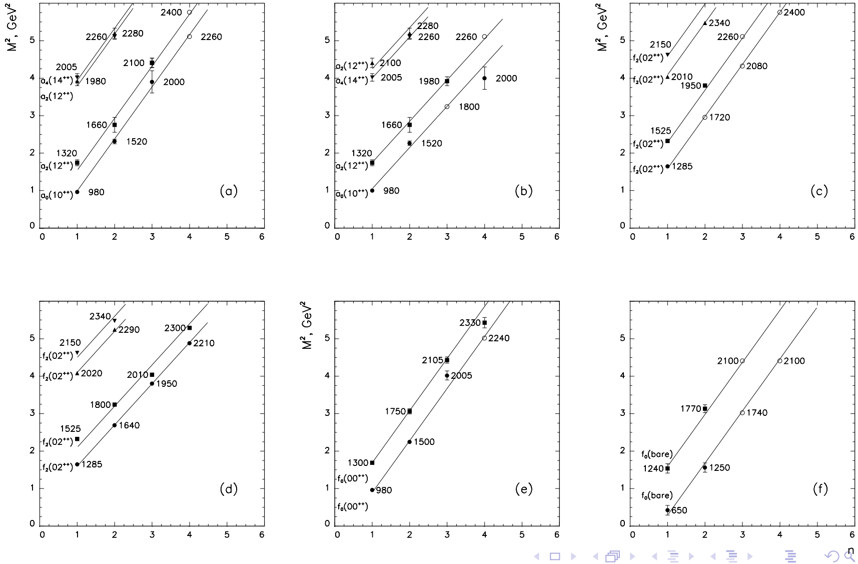
$$m^2 = m_0^2 + a'n$$

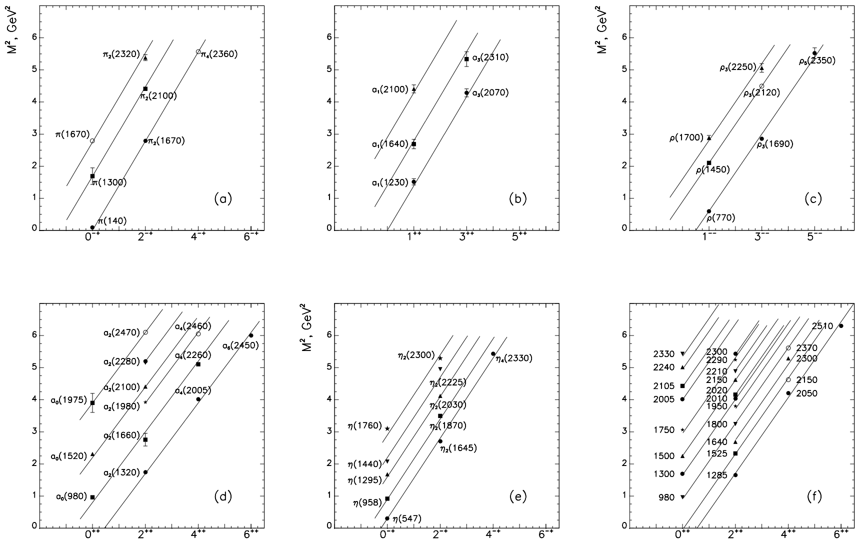
$a = 2\pi\sigma$

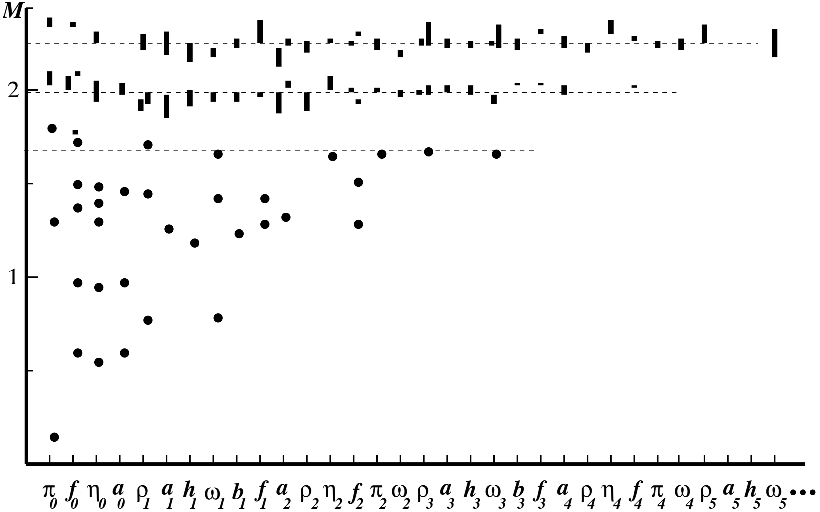
σ - string tension

$\sigma \simeq \sigma' \simeq 1$ GeV/fm

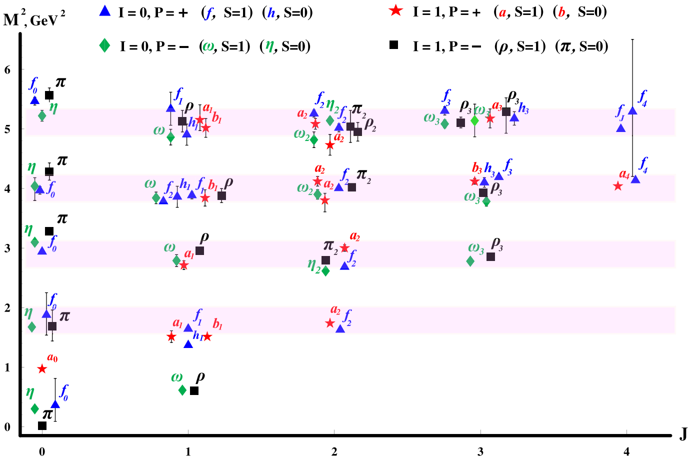
[compilation by Anisovich *et al.*, 2000]


(a)
(b)
(c)
(d)
(e)
(f)
M², GeV²
n


M², GeV²
π₂(2320)
π₄(2360)
π₂(2100)
π(1670)
π₂(1670)
π(1300)
π(140)
(a)
a₃(2310)
a₁(2100)
a₃(2070)
a₁(1640)
a₁(1230)
(b)
ρ₃(2250)
ρ₅(2350)
ρ₃(2120)
ρ(1700)
ρ₃(1690)
ρ(1450)
ρ(770)
(c)
a₂(2470)
a₄(2460)
a₆(2450)
a₂(2280)
a₄(2260)
a₀(1975)
a₂(2100)
a₄(2005)
a₂(1980)
a₂(1660)
a₀(1520)
a₂(1320)
a₀(980)
(d)
η₂(2300)
η₄(2330)
η₂(2225)
η₂(2030)
η(1760)
η₂(1870)
η₂(1645)
η(1440)
η(1295)
η(958)
η(547)
(e)
2510
2330
2300
2290
2370
2240
2210
2300
2150
2150
2105
2020
2005
2010
2050
1950
1750
1800
1640
1500
1525
1300
1285
980
(f)
J



meson states from [Glozman, 2007] – large accidental degeneracy



$q\bar{q}$ states from [Shifman, Veinshtein, 2007]

Explanation of spectra is an open question in QCD

The Dirac equation

The free spin-1/2 field is described by the Lagrangian

$$L = \bar{\psi}^{\alpha}\left(i\partial_{\mu}\gamma^{\mu}_{\alpha\beta} - m\delta_{\alpha\beta}\right)\psi^{\beta} = \bar{\psi}\left(i\partial_{\mu}\gamma^{\mu} - m\right)\psi = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi,$$

where $\alpha, \beta = 1, 2, 3, 4$ is the Dirac index and $\bar{\psi} = \psi^{\dagger}\gamma^{0}$. The γ matrices (in the Dirac representation) are:

$$\gamma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ -\sigma^{i} & 0 \end{pmatrix}$$

One also introduces

$$\beta = \gamma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; \alpha^{i} = \gamma^{0}\gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ \sigma^{i} & 0 \end{pmatrix}, \; \gamma_{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

We have the equation for the four-component Dirac spinors

$$(i\partial_\mu\gamma^\mu - m)\, q(\vec{x},t) = \left(i\gamma^0\partial_0 + i\vec{\gamma}\cdot\vec{\nabla} - m\right) q(\vec{x},t) = 0.$$

To separate time, substitute $q(\vec{x},t) = w(\vec{x})e^{-i\epsilon t}$ and multiply on the left by β. Then

$$\left(\frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta m\right) w(\vec{x}) = \epsilon w(\vec{x}).$$

Operator

$$h = \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta m$$

is called the Dirac Hamiltonian.

In the momentum representation

$$w(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} w(\vec{p}),$$

we have

$$(\vec{\alpha}\cdot\vec{p} + \beta m)\, w(\vec{p}) = \epsilon w(\vec{p}).$$

Explictly,

$$\begin{pmatrix} (m-\epsilon) & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -(m+\epsilon) \end{pmatrix} w(\vec{p}) = 0,$$

which a Hermitian eigenvalue problem for ϵ. Denote $E_p = \sqrt{\vec{p}^2 + m^2}$ and

$$\chi_{\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_{-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

One can verify with no difficulty that the vectors

$$u(\vec{p}, s) = N \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m} \end{pmatrix} \chi_s, \qquad s = \pm 1/2$$

are solutions for $\epsilon = E_p$, and vectors

$$v(-\vec{p}, s) = N \begin{pmatrix} -\frac{\vec{\sigma}\cdot\vec{p}}{E_p+m} \\ 1 \end{pmatrix} \chi_s, \qquad s = \pm 1/2$$

for $\epsilon = -E_p$. If the normalization constant is $N = \sqrt{\frac{E_p+m}{2m}}$ (covariant normalization), then $\bar{u}u = \bar{v}v = 1$

The Dirac equation

$$hq(\vec{x}) = \varepsilon q(\vec{x}),$$

can be generalized to account for the scalar spherical potential,

$$h = -i\alpha \cdot \nabla + \beta S(r).$$

The total angular momentum operator

$$\vec{J} = \vec{L} + \frac{\vec{\sigma}}{2}$$

and the operator

$$K = \beta \left(\vec{L} \cdot \vec{\sigma} + 1 \right)$$

commute with h and among themselves. Thus J^2, J_z and K may be used to classify states.
The eigenvalues of J^2 are $j(j+1)$, and the eigenvalues of K^2 are $(j+\frac{1}{2})^2$, hence the eigenvalues of K are $\pm(j+\frac{1}{2})$. This means that we only need J_z and K to classify the states, as J^2 may be expressed via K

(in the calculation above one applies the formulas $\sigma_a \sigma_b = \delta_{ab} + i\epsilon_{abc}\sigma_c$ and $\epsilon_{abc} L_a L_b = i L_c$, from where $J^2 = L^2 + \vec{L}\cdot\vec{\sigma} + \frac{3}{4}$ and $K^2 = L^2 + \vec{L}\cdot\vec{\sigma} + 1 = J^2 + \frac{1}{4}$)

Since J_z and K have the block structure

$$J_z = \begin{pmatrix} J_z & 0 \\ 0 & J_z \end{pmatrix}, \quad K = \begin{pmatrix} k & 0 \\ 0 & -k \end{pmatrix},$$

where $k = \vec{L}\cdot\vec{\sigma} + 1$, the solution of the Dirac equation which is an eigenstate of J_z and K, *i.e.*,

$$J_z q^\mu_\kappa = \mu q^\mu_\kappa, \quad K q^\mu_\kappa = -\kappa q^\mu_\kappa,$$

may be written as

$$q(\vec{x})^\mu_\kappa = \begin{pmatrix} G(r)\xi^\mu_\kappa(\theta,\varphi) \\ -iF(r)\xi^\mu_{-\kappa}(\theta,\varphi) \end{pmatrix}$$

(decomposition into the upper and lower components)

Above (r, θ, φ) are the spherical coordinates and ξ^μ_κ is an eigenstate of J_z i k:

$$J_z \xi^\mu_\kappa = \mu \xi^\mu_\kappa, \;\; k \xi^\mu_\kappa = -\kappa \xi^\mu_\kappa.$$

Furthermore, since $k^2 = L^2 + L \cdot \sigma = L^2 + k$, or $L^2 = k^2 - k$, we find

$$\begin{aligned} L^2 \xi^\mu_\kappa &= l(l+1)\xi^\mu_\kappa = \kappa(\kappa+1)\xi^\mu_\kappa \\ L^2 \xi^\mu_{-\kappa} &= l(l+1)\xi^\mu_\kappa = \kappa(\kappa-1)\xi^\mu_{-\kappa} \end{aligned}$$

When $\kappa < 0$, then for ξ^μ_κ we have $l = -\kappa - 1 = j - 1/2$, and so on. The able gives the values of l:

	ξ^μ_κ	$\xi^\mu_{-\kappa}$
$\kappa < 0$	$j - \frac{1}{2}$	$j + \frac{1}{2}$
$\kappa > 0$	$j + \frac{1}{2}$	$j - \frac{1}{2}$

Note that the upper and lower components have l differing by 1. In particular, the state $j = \frac{1}{2}$ $(\kappa = -1)$ has $l = 0$ for the upper component and $l = 1$ for the lower component.

One can also recognize

$$\xi_\kappa^\mu = \sum_{s_z=\pm\frac{1}{2}} \langle l, \mu - s_z, \frac{1}{2}, s_z | j, \mu \rangle Y^l_{\mu - s_z} | \frac{1}{2}, s_z \rangle,$$

where $\langle l, \mu - s_z, \frac{1}{2}, s_z | j, \mu\rangle$ is a Clebsha-Gordan coefficient, $Y^l_{\mu-s_z}$ a spherical harmonic, and $|\frac{1}{2}, s_z\rangle$ a two-component spinor of spin projection s_z. In particular,

$$\xi^\mu_{-1} = |\frac{1}{2}, \mu\rangle$$

Let us denote $\hat{r} = \vec{x}/r$. Using $(\sigma \cdot \hat{r})^2 = 1$ and the fact that $\sigma \cdot \hat{r}$ is pseudoscalar, we obtain the identity $\sigma \cdot \hat{r} \xi^\mu_\kappa = \eta \xi^\mu_{-\kappa}$, where η is a phase factor, $|\eta| = 1$. In the Condon-Shortley convention $\eta = -1$. We may now rewrite

$$q^\mu_\kappa = \begin{pmatrix} G \\ i\sigma \cdot \hat{r} F \end{pmatrix} \xi^\mu_\kappa$$

Parity of the spinor ξ^{μ}_{κ} is $(-1)^{l} = \mathrm{sgn}(\kappa)\,(-1)^{\kappa}$, while

$$Pq^{\mu}_{\kappa} = \eta_P \mathrm{sgn}(\kappa)\,(-1)^{\kappa}\, q^{\mu}_{\kappa},$$

where η_P is a phase factor. The most common convention is $\eta_P = 1$ (for the Dirac spinor the parity operator is $P = \eta_P \gamma_0 \times [\vec{r} \leftrightarrow -\vec{r}]$)
Applying the identity

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} - i\frac{\hat{r}}{r} \times \vec{L}$$

one gets that

$$-i\vec{\alpha}\cdot\vec{\nabla} = -i\vec{\alpha}\cdot\hat{r}\frac{\partial}{\partial r} + i\frac{1}{r}\vec{\alpha}\cdot\hat{r}(\beta K - 1)$$

Finally, the Dirac equation for the radial functions G and F has the form

$$\begin{aligned} \frac{dF}{dr} &= \frac{\kappa - 1}{r}F + (\varepsilon - S)G, \\ \frac{dG}{dr} &= -\frac{1+\kappa}{r}G - (\varepsilon + S)F \end{aligned}$$

The Bogolyubov bag

Consider a quark (Dirac particle) in a scalar spherical potential

$$S(r) = \begin{cases} 0 & \text{dla} \quad r < R \\ M & \text{dla} \quad r \geq R \end{cases} . \tag{2}$$

This corresponds to a massless particle inside the sphere of radius R, and a particle of mass M outside. The lowest-energy state has $j^P = \frac{1}{2}^+$ is described by the spinor ($\kappa = -1$)

$$q(\vec{r}, s) = \begin{pmatrix} G(r) \\ i\sigma \cdot \hat{r} F(r) \end{pmatrix} \chi(s), \quad s = \pm\frac{1}{2}.$$

The radial functions G and F satisfy the first-order differential equations

$$\begin{aligned} \frac{dG}{dr} &= -(\varepsilon + S(r))\, F, \\ \frac{dF}{dr} &= -\frac{2}{r} F + (\varepsilon - S(r))\, G \end{aligned}$$

The function $H(r) = rG(r)$ satisfies the equation

$$\begin{aligned} \left[\frac{d^2}{dr^2} + \varepsilon^2\right] H(r) &= 0 \quad \text{dla} \quad r < R, \\ \left[\frac{d^2}{dr^2} - \left(M^2 - \varepsilon^2\right)\right] H(r) &= 0 \quad \text{dla} \quad r \geq R \end{aligned}$$

Because we'll take $M \to \infty$, we assume $M > \varepsilon$. G and F must be regular at $r = 0$ and $r \to \infty$, H vanishes for $r = 0$ and $r \to \infty$. Thus

$$H(r) = \begin{cases} A \sin \varepsilon r & \text{for} \quad r < R \\ B e^{-\sqrt{M^2 - \varepsilon^2} r} & \text{for} \quad r \geq R \end{cases}$$

For functions G and F we get

$$\begin{aligned} G(r) &= \begin{cases} \frac{A}{r} \sin \varepsilon r & \text{for} \quad r < R \\ \frac{B}{r} e^{-\sqrt{M^2 - \varepsilon^2} r} & \text{for} \quad r \geq R \end{cases}, \\ F(r) &= \begin{cases} \frac{A}{\varepsilon r}\left(-\frac{\sin \varepsilon r}{r} + \varepsilon \cos \varepsilon r\right) & \text{for} \quad r < R \\ \frac{B}{(M+\varepsilon) r}\left(-\frac{1}{r} - \sqrt{M^2 - \varepsilon^2}\right) e^{-\sqrt{M^2 - \varepsilon^2} r} & \text{for} \quad r \geq R \end{cases} \end{aligned}$$

The above equations are of first order, thus their regular solutions must be continuous functions, which leads to the conditions

$$\lim_{r\to R^-} G(r) = \lim_{r\to R^+} G(r),$$
$$\lim_{r\to R^-} F(r) = \lim_{r\to R^+} F(r),$$

or explicitly,

$$\frac{A\sin\varepsilon R}{R} = \frac{B}{R} e^{-\sqrt{M^2-\varepsilon^2}R}$$
$$\frac{A}{\varepsilon}\left(-\frac{\sin\varepsilon R}{R^2} + \frac{\varepsilon}{R}\cos\varepsilon R\right) = \frac{B}{\varepsilon+M}\left(\frac{-1}{R^2} - \frac{\sqrt{M^2-\varepsilon^2}}{R}\right) e^{-\sqrt{M^2-\varepsilon^2}R}.$$

This homogeneous set of equations has a solution when

$$\cos\varepsilon R + \frac{\sqrt{M^2-\varepsilon^2}}{M+\varepsilon}\sin\varepsilon R = \frac{M}{(M+\varepsilon)\,\varepsilon R}\sin\varepsilon R$$

Note this is a quantization condition for ε

Next we take the limit of the infinite wall, $M \to \infty$, in order to “confine” the quarks. Then G and F vanish outside the bag, and inside have the form

$$G = N\frac{\sin \varepsilon r}{\varepsilon r} \equiv N j_0(\varepsilon r), \qquad F = N\frac{\sin \varepsilon r - \varepsilon r \cos \varepsilon r}{(\varepsilon r)^2} \equiv N j_1(\varepsilon r),$$

where N is a normalization constant. The continuity condition is

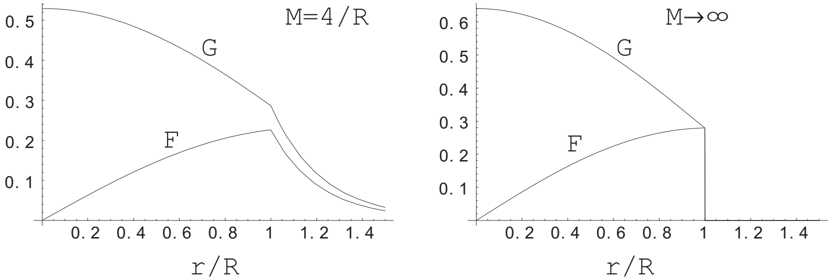
$$j_0(\varepsilon R) = j_1(\varepsilon R),$$

which is equivalent to $G(R) = F(R)$. This equation for ε may be solved numerically. Denoting $\varepsilon = \omega/R$, we find

$$\omega_1 \simeq 2.04 \quad , \omega_2 \simeq 5.40, \quad \dots$$

The continuity condition may also be written in the form

$$i\vec{\gamma} \cdot \hat{r} q(R) = q(R)$$



The baryon density is

$$\rho = q^\dagger q = N^2[j_0(\frac{\omega}{R}r)^2 + j_1(\frac{\omega}{R}r)^2]$$

and does not vanish at $r \to R^-$. The probability currentis $\vec{j} = \bar{q}\vec{\gamma}q$ and its normal component to the surface of the bag, $\hat{r} \cdot \vec{j} = \bar{q}\hat{r} \cdot \vec{\gamma}q$, vanishes identically. This means that the quarks do not leak out from the bag.

The energy-momentum tensor of a free massless Dirac particle is $T_q^{\mu\nu} = \frac{i}{2}\bar{q}\gamma^\mu \overset{\leftrightarrow}{\partial}{}^\nu q$, where $\overset{\leftrightarrow}{\partial}{}^\nu = \overrightarrow{\partial^\nu} - \overleftarrow{\partial^\nu}$. Conservation of energy and momentum yields $\partial_\mu T^{\mu\nu} = 0$. This holds for instance for the massless free particle, since from the Dirac equation $\partial_\mu \gamma^\mu q = 0$. In the Bogolyuobov bag there are complications at $r = R$, because

$$T_{\text{Bog}}^{\mu\nu} = \left(\frac{i}{2}\bar{q}\gamma^\mu \overset{\leftrightarrow}{\partial}_\nu) q\right) \Theta(R-r),$$

where $\Theta(x) = 1$ denotes the step function. Using the formula $\nabla_i \Theta(R-r) = -\hat{r}_i \delta(R-r)$ we find that

$$\partial_\mu T_{\text{Bog}}^{\mu\nu} = -\frac{1}{2}\delta(R-r)\partial^\nu(\overline{q}q) \neq 0 \qquad \text{dla} \quad \nu \neq 0.$$

This means, that the energy-momentum tensor is violated at the surface of the bag.

The solution is the MIT bag, which adds the term to the Lagrangian of the form $-B\theta(R-r)$. Then

$$\partial_\mu T^{\mu\nu}_{\mathrm{MIT}} = \delta(R-r)\left(-\frac{1}{2}\hat{r}\cdot\partial(\overline{q}q) - B\right)\hat{r}^\nu,$$

which vanishes, if $B = -\frac{1}{2}n\cdot\partial(\overline{q}q)$. For the baryon in the ground state ($N_c = 3$ $\kappa = -1$ quarks) this gives

$$B = \frac{3}{4\pi R^3}\frac{\omega_1}{R}.$$

The same result follows from the *energetic stability* of the bag with the change of R. The nucleon energy is

$$E_N(R) = \frac{3\omega_1}{R} + \frac{4\pi}{3}R^3 B$$

(kinetic energy of quarks + energy to dig the hole). The energetic stability condition $\frac{dE(R)}{dR} = 0$ gives the equation for B. We can then write

$$E_N(B) = \frac{4}{3}(3\omega_1)^{3/4}(4\pi B)^{1/4}.$$

Fixing the parameters to the average nucleon and $\Delta(1232)$ mass yields

$$B^{1/4} = 111\mathrm{MeV}, \quad R_N = 1.48\mathrm{fm}$$

(too large radius). For the ground state mesons

$$E_{mes}(R) = \frac{2\omega_1}{R} + \frac{4\pi}{3}R^3 B, \tag{3}$$

which numerically gives $E_{mes} = 801$MeV, close to the ρ and ω masses and $R_{mes} = 1.34$ fm.

The following extensions were introduced: one-gluon exchange, mass of the strange quark, center-of-mass motion correction.

The gluon exchange leads to the splitting of N and Δ, similarly as in NRQM:

$$\Delta E_g = \frac{\alpha_s}{R}\sum_{i<j} f(m_i, m_j, R)\vec{\sigma}_i \cdot \vec{\sigma}_j \times \left\{ \begin{array}{ll} 1 & \text{for baryons} \\ 2 & \text{for mesons} \end{array} \right. ,$$

where $f(m_i, m_j, R)$ are certain known functions of the quark masses and the bag radius

The introduction of m_s is straightforward. Best fits give $m_s \sim 300$ MeV, larger than the current mass. One can show that ω_1 grows form the value 2.04 for $m_s = 0$ to π for $m_s \to \infty$.
The center-of-mass corrections result form the lack of the separation of the center-of-mass motion. This plagues all models which violate the translational symmetry. An approximate formula gives $\Delta E_{CM} = -Z/R \sim -3/8\, \omega/R$. With all corrections included, the energy becomes

$$M(R) = \frac{\sum_i \omega(i)}{R} + \frac{4\pi}{3} R^3 B + \Delta E_g - \frac{Z}{R}.$$

Spectroscopy is generally quite good, except for the pion (and there is a good reason for it!)

The axial charge of the nucleon (measured in weak decays) is defined as

$$g_A = 2\langle p\uparrow | \int d^3r \; j^3_{5,3} | p\uparrow\rangle = \langle p\uparrow | \int d^3r \; \sum_i q_i^\dagger \sigma_3 \tau_3 q_i | p\uparrow\rangle,$$

which gives

$$g_A = \frac{5}{3}\left(1 - \frac{1}{3}\frac{2\omega_1 - 3}{\omega_1 - 1}\right) \approx 1.1$$

MIT bag:	$g_A \approx 1.1$
$SU(3)_F$	$g_A = \frac{5}{3} g_A^q$
exp.:	$g_A = 1.27$

The Roper resonance N(1440) i s a radial excitation of the nucleon. In the MIT bag we get it by exciting one quark radially.

$$M_{\text{Roper}} = \frac{4}{3}(2\omega_1 + \omega_2)^{3/4}(4\pi B)^{1/4},$$

which gives $M_{\text{Roper}}/M_N = 1.39$ (exp.: 1.53)

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Emmy Noether: for each continuous symmetry of the Lagrangian there is an associated conserved current

Examples: translations in time and space – energy and momentum, rotations – angular momentum, $U(1)$ phase transformation of quarks – baryon current

Internal symmetries: consider the Lagrangian with n real scalar fields, $n > 1$,

$$L = L(\{\partial_\mu \phi_i(x)\}, \{\phi_i(x)\}).$$

It implies the Euler-Lagrange equations

$$\partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi_i)} - \frac{\partial L}{\partial \phi_i} = 0.$$

Infinitesimal *global internal transformation* (gauge transformations of the first kind) is defined as $\phi_i(x) \to \phi_i(x) + \delta\phi_i(x)$, where

$$\delta\phi_i(x) = \varepsilon f_i(\{\phi_j(x)\}),$$

where ε is small and independent of x and f_i are functions of $\{\phi_j(x)\}$.

We say that the system possesses a symmetry, if L is invariant under its action. The Noether current is defined as

$$j^\mu(x) = \frac{\partial L}{\partial(\partial_\mu \phi_i)} f_i(\{\phi_j(x)\}).$$

The Noether theorem for the global internal symmetry: if the system has the above-defined symmetry, then $\partial_\mu j^\mu(x) = 0$, *i.e* the Noether current is conserved

Proof: L is invariant, hence $\delta L = 0$, or

$$\delta L = \frac{\partial L}{\partial(\partial_\mu \phi_i)} \partial_\mu \delta\phi_i + \frac{\partial L}{\partial \phi_i} \delta\phi_i = 0,$$

and

$$\begin{aligned} \partial_\mu j^\mu &= \left(\partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi_i)} \right) f_i + \frac{\partial L}{\partial(\partial_\mu \phi_i)} \partial_\mu f_i = \\ &= \left(\partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi_i)} - \frac{\partial L}{\partial \phi_i} \right) f_i = 0, \end{aligned}$$

where the last equality follows from the Euler-Lagrange equations.

In many applications only a part of L is symmetric. Denote $L = L_0 + L_1$, where L_0 is symmetric L_1 is not. Such a breaking of the symmetry is called explicit. In this case

$$\partial_\mu j^\mu = \frac{\partial L_1}{\partial(\partial_\mu \phi_i)} \partial_\mu f_i + \frac{\partial L_1}{\partial \phi_i} f_i \neq 0$$

Example:

$$L = \frac{1}{2}\left(\partial_\mu S(x)\right)^2 + \frac{1}{2}\left(\partial_\mu P(x)\right)^2 - \frac{m^2}{2}\left(S(x)^2 + P(x)^2\right)$$

E-L equations:

$\left(\partial^\mu \partial_\mu + m^2\right) S(x) = 0$, $\left(\partial^\mu \partial_\mu + m^2\right) P(x) = 0$

Rotational symmetry in $S - P$ space:
$\delta S(x) = -\varepsilon P(x)$, $\delta P(x) = \varepsilon S(x)$, $f_S = -P$, $f_P = S$

$$\delta L = \left(\partial_\mu \delta S\right)\left(\partial_\mu S\right) + \left(\partial_\mu \delta P\right)\left(\partial_\mu P\right) - m^2\left(S\delta S + P\delta P\right) =$$
$$= -\varepsilon\left(\partial_\mu P\right)\left(\partial_\mu S\right) + \varepsilon\left(\partial_\mu S\right)\left(\partial_\mu P\right) - m^2\left(-\varepsilon PS + \varepsilon SP\right) = 0$$

Noether current:
$j^\mu = S\partial_\mu P - P\partial_\mu S$

$\partial_\mu j^\mu = S\partial_\mu \partial^\mu P - P\partial_\mu \partial^\mu S = Sm^2 P - Pm^2 S = 0$

When we add above a symmetry-breaking term, *e.g*, $L_1 = -cS(x)$, then

$$\begin{aligned}\left(\partial^\mu \partial_\mu + m^2\right) S &= c, \\ \partial_\mu j^\mu &= -cm^2 P.\end{aligned}$$

The charge is defined as $Q = \int d^3 x j^0(x)$. If the symmetry is preserved, then

$$\frac{d}{dt} Q = \int d^3 x \frac{d}{dt} j^0(x) = -\int d^3 x \vec{\nabla} \cdot \vec{j}(x) = -\int_S d\vec{n} \cdot \vec{j}(x) = 0,$$

where tha last equality follows from a fast fall-off of j. Thus the charge is constant in time.
The symmetry transformations form a group. In quantum field theory the charges Q^a, $a = 1, ..., n$ satisfy the charge algebra:

$$[Q^a, Q^b] = i f^{ab}_{\ \ ..c} Q^c,$$

where $f^{ab}_{\ \ ..c}$ are the structure constants of the group.

[show movies]
Definition: If the Lagrangian of a theory possesses a group of global symmetries, while the ground state has a lower group of symmetries, we call this phenomenon the spontaneous symmetry breaking (hidden symmetry, Nambu-Goldstone phase).

Goldstone's theorem: If the group of continuous global symmetries of the Lagrangian has n_L generators, denoted as T^a, while the ground state $|\mathrm{vac}\rangle$ (vacuum) has a lower symmetry, *i.e.* there exist n such T^a and j, for which $T^a_{ij}\langle \mathrm{vac}|\phi^j|\mathrm{vac}\rangle \neq 0$, where ϕ^j are fields of spin 0, then there exist n massless spin-0 bosons. Their quantum numbers are the same as of generators T^a. These bosons are called the Goldstone bosons, and the whole phenomenon the Goldstone mechanism.

Example for classical fields

$$L = \frac{1}{2}\left(\partial_\mu S\right)^2 + \frac{1}{2}\left(\partial_\mu P\right)^2 - V(S,P), \quad V(S,P) = \frac{\mu}{2}\left(S^2 + P^2\right) + \frac{\lambda^2}{4}\left(S^2 + P^2\right)^2$$

Assume $\lambda^2 > 0$, but the sign of μ is positive or negative. L is symmetric with respect to rotations in the $S - P$ space:

$$S \to \cos\theta\, S - \sin\theta\, P, \quad P \to \sin\theta\, S + \cos\theta\, P$$

The ground state is a state of lowest energy, $S(x) = S$ and $P(x) = P$ minimize the potential

$$\left.\frac{\partial V(S,P)}{\partial S}\right|_{S=S_0, P=P_0} = 0, \quad \left.\frac{\partial V(S,P)}{\partial P}\right|_{S=S_0, P=P_0} = 0,$$

which explicitly gives the conditions

$$\mu S_0 + \lambda^2 S_0\left(S_0^2 + P_0^2\right) = 0, \quad \mu P_0 + \lambda^2 P_0\left(S_0^2 + P_0^2\right) = 0$$

Case $\mu > 0$: the Wigner phase

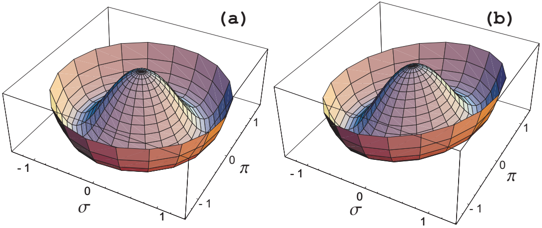
$$S_0 = 0, \qquad P_0 = 0.$$

This is the so called trivial vacuum or the Wigner phase.
Excitations of S and P have the mass $\sqrt{\mu}$, since

$$S = S_0 + \delta S,\ P = P_0 + \delta P \ \rightarrow\ \tfrac{1}{2}\mu(\delta S^2 + \delta P^2)$$

[show movie mexhat2.avi]

Case $\mu < 0$: the Nambu-Goldstone phase



$$S_0 = \frac{\sqrt{-\mu}}{\lambda} \cos \alpha, \qquad P_0 = \frac{\sqrt{-\mu}}{\lambda} \sin \alpha,$$

where α is any angle. Because of the symmetry, we may assign it any value without a loss of generality, *e.g* $\alpha = \pi$, which gives

$$P_0 = 0, \qquad S_0 = -\frac{\sqrt{-\mu}}{\lambda}$$

[show movies mexhat3.avi, mexhat4.avi]

The vacuum is not symmetric with respect to the symmetry transformation. According to the Goldstone theorem a massless excitation appears. Indeed, substitution of $S = S_0 + \delta S$, $P = P_0 + \delta P$ yields

$$\begin{aligned} V &= \frac{\mu}{2}\left(S_0^2 + 2S_0\delta S + \delta S^2 + \delta P^2\right) + \frac{\lambda^2}{4}\left(S_0^2 + 2S_0\delta S + \delta S^2 + \delta P^2\right)^2 = \\ &= \frac{\mu}{2}S_0^2 + \frac{\lambda^2}{4}S_0^4 + \left(\mu S_0 + \lambda^2 S_0^3\right)\delta S + \frac{1}{2}\left(\mu + 3\lambda^2 S_0\right)\delta S^2 \\ &+ \frac{1}{2}\left(\mu + \lambda^2 S_0\right)\delta P^2 = \mathrm{const} - \mu\delta S^2 + \dots \end{aligned}$$

The coefficient of δP^2 vanishes, hence the excitation of P is massless. The mass of the excitation of S is $M_S = \sqrt{-2\mu}$

The Lagrangian of QCD

$$L = \bar{\psi}(D - m)\psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$
$$= \bar{u}(D - m_u)u + \bar{d}(D - m_u)d + \bar{s}(D - m_u)s + \ldots - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a,$$

has internal exact global symmetries. These symmetries to not transform the gluons, while quark fields transform via a phase change:

$$U(1)_V \quad : \qquad u \to e^{i\alpha} u, \qquad j^\mu_B = \bar{u}\gamma^\mu u, \qquad \text{(flavor u)}$$
$$U(1)_V \quad : \qquad d \to e^{i\alpha} d, \qquad j^\mu_B = \bar{d}\gamma^\mu d, \qquad \text{(flavor d)}$$
$$\ldots$$

The symmetries mean the conservation of each current. They may be combined in linear combinations →

$$U(1)_V \ (\mathrm{B}) \quad : \quad \psi \to e^{i\alpha}\psi, \qquad j_B^\mu = \frac{1}{3}\bar{\psi}\gamma^\mu\psi = \frac{1}{3}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d + \dots\right),$$

$$U(1)_V \ \ (I_3) \quad : \quad \psi \to e^{i\alpha\tau_3/2}\psi, \qquad j_3^\mu = \bar{\psi}\gamma^\mu\frac{\tau_3}{2}\psi = \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right),$$

$$U(1)_V \ (\mathrm{s}) \quad : \quad \psi \to e^{i\alpha(\lambda_8-1)/\sqrt{3}}\psi, \qquad j_s^\mu = \bar{\psi}\gamma^\mu(\lambda_8-1)/\sqrt{3}\psi = \bar{s}\gamma^\mu s,$$

$$\dots$$

A very good symmetry, broken explicitly very weakly by the difference of masses of the u and d quarks, is the group of the isospin rotations (the rotation about the 3rd axis is an exact symmetry):

$$SU(2)_V \quad : \quad \psi \to e^{i\vec{\alpha}\cdot\vec{\tau}/2}\psi, \qquad j_a^\mu = \bar{\psi}\gamma^\mu\frac{\tau_a}{2}\psi,$$

$$\partial_\mu j_a^\mu \quad = \quad \frac{m_u - m_d}{2}\bar{\psi}\frac{[\tau_a, \tau_3]}{2i}\psi, \qquad \text{(isospin)}$$

Extension to three flavors is a worse symmetry, due to a larger value of the strange quark mass:

$$
\begin{aligned}
SU(3)_V \quad &: \qquad \psi \to e^{i\alpha_a \lambda^a/2}\psi, \qquad j_a^\mu = \bar{\psi}\gamma^\mu \frac{\lambda_a}{2}\psi, \\
\partial_\mu j_a^\mu \quad &= \quad \frac{m_u - m_d}{2}\bar{\psi}\frac{[\lambda_a, \lambda_3]}{2i}\psi - \frac{m_s}{\sqrt{3}}\bar{\psi}\frac{[\lambda_a, \lambda_8]}{2i}\psi
\end{aligned}
$$

A quite good symmetry is the axial symmetry:

$$
\begin{aligned}
SU(2)_A \quad &: \qquad \psi \to e^{i\gamma_5 \vec{\alpha}\cdot\vec{\tau}/2}\psi, \qquad j_{5,a}^\mu = \bar{\psi}\gamma^\mu\gamma_5 \frac{\tau_a}{2}\psi, \\
\partial_\mu j_{5,a}^\mu \quad &= \quad \frac{m_u + m_d}{2}\bar{\psi}\tau_a i\gamma_5\psi + \frac{m_u - m_d}{2}\delta_{a3}\bar{\psi}i\gamma_5\psi
\end{aligned}
$$

The isospin and axial charges

$$\begin{aligned} Q^a(t) &= \int d^3x j_0^a(\vec{x},t) = \int d^3x \psi^\dagger \frac{\tau_a}{2}\psi \\ Q_5^a(t) &= \int d^3x j_{0,5}^a(\vec{x},t) = \int d^3x \psi^\dagger \gamma_5 \frac{\tau_a}{2}\psi \end{aligned}$$

satisfy the commutation relations

$$\begin{aligned} [Q^a(t), Q^b(t)] &= i\varepsilon^{abc} Q^c(t) \\ [Q_5^a(t), Q_5^b(t)] &= i\varepsilon^{abc} Q^c(t) \\ [Q^a(t), Q_5^b(t)] &= i\varepsilon^{abc} Q_5^c(t) \end{aligned}$$

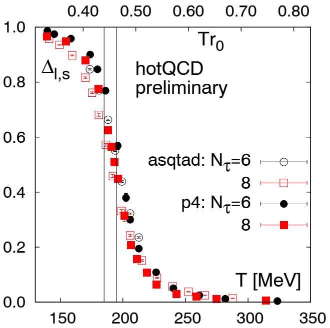
They follow from the canonical commutation rules for the quark fields: $\{\psi(x), \psi^\dagger(y)\}_{ET} = \delta^3(\vec{x}-\vec{y})$. The left-right combinations, $Q_{R,L}^a = (Q^a \pm Q_5^a)$, fulfill the chiral $SU(2)_L \otimes SU(2)_R$ algebra

$$\begin{aligned} [Q_R^a, Q_R^b] &= i\varepsilon^{abc} Q_R^c \\ [Q_L^a, Q_L^b] &= i\varepsilon^{abc} Q_L^c \\ [Q_R^a, Q_L^b] &= 0 \end{aligned}$$

Quark condensate

In the σ model the S field acquires non-zero expectation value in the Goldstone phase. In QCD the equivalent is the chiral condensate $\langle\bar{\psi}\psi\rangle \neq 0$. Estimates from theoretical studies show that

$$\langle\bar{\psi}\psi\rangle \sim -(250\ \text{MeV})^3$$



Empirical facts for the Goldstone phase

- Small mass of the pion: pion is a pseudo-Goldstone boson: $m_\pi = 140$ MeV, $m_\pi^2/m_\rho^2 \simeq 0.03$
- Non-existance of parity-doublet states. Assume exact chiral symmetry, $[H, Q_5^a] = 0$ and consider hadron states $|N^+\rangle$ and $|N_a^-\rangle = Q_5^a|N^+\rangle$. They have opposite parities, as $\{P, Q_5^a\} = 0$. Let $|N^+\rangle$ have mass M, hence $H|N^+\rangle = M|N^+\rangle$. Then $Q_5^a H|N^+\rangle = Q_5^a M|N^+\rangle$ and $HQ_5^a|N^+\rangle = Q_5^a M|N^+\rangle$, therefore $H|N_a^-\rangle = M|N_a^-\rangle$. This means $|N^+\rangle$ and $|N_a^-\rangle$ are degenerate. Because the chiral symmetry is slightly broken explicitly, these would have to be approximately degenerate. Experimental spectrum does not have this feature! In the Goldstone phase there is a way out. The state $|N_a^-\rangle$ is the state $|N^+ + \mathrm{pions}\rangle$. Since the pion is massless, the masses of $|N^+\rangle$ and $|N^+ + \mathrm{piony}\rangle$ are equal

- Goldberger–Treiman relation $g_A M_N = g_{\pi NN} F_\pi + \mathcal{O}(m_\pi^2/\Lambda_\chi^2)$ (correction $\mathcal{O}(m_\pi^2/m_\rho^2)$ is only a few percent)
- S-wave scattering lengths in low-energy $\pi - N$ scattering in isospin $1/2$ and $3/2$ channels (Weinberg):

$$\begin{aligned} a_{1/2} &= \frac{m_\pi}{4\pi F_\pi^2} \simeq 0.18/m_\pi \qquad \text{exp.}: (0.171 \pm 0.005)/m_\pi \\ a_{3/2} &= -\frac{m_\pi}{8\pi F_\pi^2} \simeq -0.09/m_\pi \qquad \text{exp.}: -(0.088 \pm 0.004)/m_\pi \end{aligned}$$

- Scattering lengths in $\pi\pi$ scattering in isospin 0 and 2 channels:

$$\begin{aligned} a_0 &= \frac{7m_\pi}{32\pi F_\pi^2} \simeq 0.16/m_\pi \qquad \text{exp.}: (0.26 \pm 0.05)/m_\pi \\ a_2 &= -\frac{2m_\pi}{32\pi F_\pi^2} \simeq -0.046/m_\pi \qquad \text{exp.}: -(0.028 \pm 0.012)/m_\pi \end{aligned}$$

- Adler-Weisberger: $g_A^2 - 1 = \int \frac{d\omega}{\omega^2} \sqrt{\omega^2 - m_\pi^2} \left(\sigma_{\pi^+ p}(\omega) - \sigma_{\pi^- p}(\omega)\right)$
- Soft pion theorems, chiral perturbation theory

CVC and PCAC

As stated abave, for $m_u = m_d \equiv \bar{m}$

$$\partial_\mu j^\mu_a = 0,$$

which is termed CVC - conservation of vector current, and

$$\partial_\mu j^\mu_{5,a} = \bar{m}\bar{\psi}\tau_a i\gamma_5\psi,$$

called PCAC - partial conservation of axial currents. Currents j^μ_a i $j^\mu_{5,a}$ couple to photons and weak bosons and their matrix elements may be determined from appropriate electromagnetic and weak processes. In particular

$$\langle 0|j^\mu_{5,a}(x)|\pi_b(q)\rangle = iF_\pi q^\mu e^{-iq\cdot x}\delta_{ab},$$

where the constant $F_\pi \approx 93$ MeV is known from the decay $\pi^+ \to \mu^+\nu_\mu$. It thus follows that

$$\langle 0|\partial_\mu j^\mu_{5,a}(x)|\pi_b(q)\rangle = F_\pi q_\mu q^\mu e^{-iq\cdot x}\delta_{ab} = F_\pi m^2_\pi e^{-iq\cdot x}\delta_{ab}.$$

One introduces the pion interpolating field

$$\phi_a(x) = \frac{\partial_\mu j^\mu_{5,a}(x)}{F_\pi m_\pi^2},$$

with the property $\langle 0|\phi_a(x)|\pi_b(q)\rangle = e^{-iq\cdot x}\delta_{ab}$.
Example of a PCAC calculation: consider the propagator

$$\begin{aligned}\frac{1}{3}\sum_a \int d^4x e^{-iq\cdot x}\langle 0|T\left(\partial_\mu j^\mu_{5,a}(x), \partial_\nu j^\nu_{5,a}(0)\right)|0\rangle &= \\ \frac{1}{3}F_\pi^2 m_\pi^4 \sum_a \int d^4x e^{-iq\cdot x}\langle 0|T\left(\phi_a(x), \phi_a(0)\right)|0\rangle &= \frac{iF_\pi^2 m_\pi^4}{q^2 - m_\pi^2} f(q^2).\end{aligned}$$

Assume $f(q^2)$ is smooth near $q^2 = 0$, *i.e.* $f(0) \sim f(m_\pi^2) \equiv 1$. Next take the soft pion limit $\vec{q} \to 0$ and then $q_0 \to 0$. As a result the LHS after integration by parts becomes

$$i\frac{1}{3}\sum_a \langle 0|[Q_5^a, [Q_5^a, \mathcal{H}(0)]]|0\rangle$$

In QCD

$$\frac{1}{3}\sum_a [Q_5^a, [Q_5^a, \mathcal{H}(0)]] = \bar{m}\bar{\psi}(0)\psi(0).$$

On the RHS the soft pion limit gives

$$\bar{m}\langle 0|\bar{\psi}\psi|0\rangle = -F_\pi^2 m_\pi^2 f(0).$$

From PCAC, $f(0) \approx f(m_\pi^2) = 1$, we get

$$\bar{m}\langle\bar{\psi}\psi\rangle = -F_\pi^2 m_\pi^2.$$

This is the famous Gell-Mann-Oaks-Renner relation. We may write more generally

$$2\bar{m}\langle\bar{q}q\rangle = -F_\pi^2 m_\pi^2 + \mathcal{O}(m_\pi^4/\Lambda_\chi^4)$$

Note, that $m_\pi \sim \sqrt{\bar{m}}$ (here $\langle\bar{q}q\rangle = \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \frac{1}{2}\langle\bar{\psi}\psi\rangle$)

For three flavors $(m_s + \bar{m})\langle \bar{q}q \rangle = -F_\pi^2 m_K^2 + \mathcal{O}(m_s^2)$, which leads to

$$\frac{2\bar{m}}{\bar{m} + m_s} = \frac{m_\pi^2}{m_K^2} + \mathcal{O}(m_s^2) \simeq \frac{140^2}{494^2} \sim 0.08$$

With $\bar{m} = 6$ MeV we get from here $m_s \simeq 150$ MeV. More precisely,

$$\frac{m_d}{m_s} = \frac{m_{K^0}^2 + m_{\pi^\pm}^2 - m_{K^\pm}^2}{m_{K^0}^2 + m_{K^\pm}^2 - m_{\pi^\pm}^2} \simeq 0.050$$

$$\frac{m_u}{m_s} = \frac{2m_{\pi^0}^2 - m_{K^0}^2 - m_{\pi^\pm}^2 + m_{K^\pm}^2}{m_{K^0}^2 + m_{K^\pm}^2 - m_{\pi^\pm}^2} \simeq 0.027$$

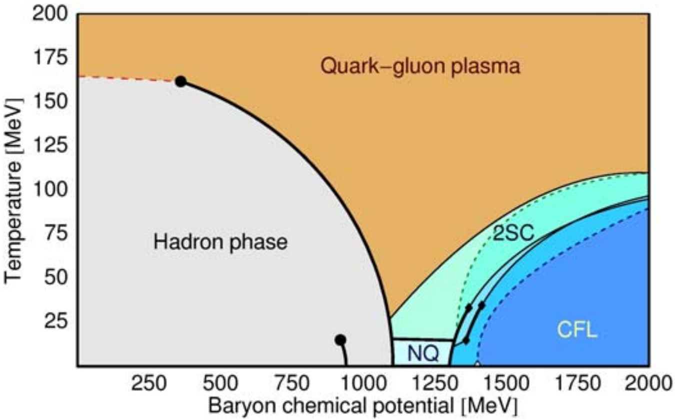
Assuming particular flavor symmetry breaking, one derives the Gell-Mann-Okubo approximate mass formulas, *e.g*

$$3m_\eta^2 + 2m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2m_{K^\pm}^2 + 2m_{K^0}^2$$

Summary of chiral symmetry breaking

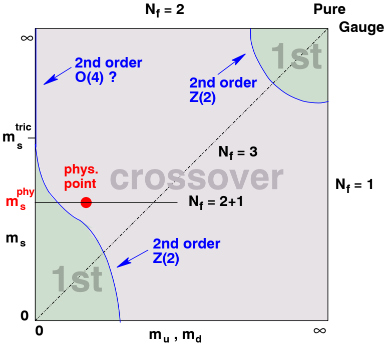
Wigner phase	Nambu-Goldstone phase
$\langle\overline{\psi}\psi\rangle = 0$	$\langle\overline{\psi}\psi\rangle \neq 0$
$m_\pi > 0$	$m_\pi = 0$
$F_\pi = 0$	$F_\pi > 0$
degenerate multiplets of opposite parity: $m_\pi = m_\sigma,\, m_\rho = m_{A_1},\, \ldots$	no degenerate multiplets of opposite parity: $m_\pi < m_\sigma,\, m_\rho < m_{A_1},\, \ldots$

Phase diagram of QCD



(information from lattice QCD and models)

Nature of the phase transition



(information from lattice QCD)

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6 Chiral models
- Chiral bags
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- Dynamical symmetry breaking, NJL model

Discussed earlier bag models break the chiral symmetry. In the Bogolyubov bag it is the term $\bar{\psi}(x)S(x)\psi(x)$. The Lagrangian of the MIT bag

$$L_{MIT}(x) = \left[\frac{i}{2}\bar{\psi}(x) \overleftrightarrow{\partial}_\mu \gamma^\mu \psi(x) - B\right] \Theta(R-r) - \frac{1}{2}\bar{\psi}(x)\psi(x)\delta(R-r)$$

leads to $\partial_\mu j^\mu_{5,a}(x) = -\frac{1}{2}\bar{\psi}(x)\tau_a i\gamma_5\psi(x)\delta(r-R)$, hence the axial current is broken at the surface of th bag. Chodos and Thorn proposed the generalization:

$$L_{CT}(x) = \left[\frac{i}{2}\bar{\psi} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi - B\right] \Theta(R-r)$$
$$-\frac{1}{2\sqrt{\sigma^2+\pi^2}}\bar{\psi}\left(\sigma + i\gamma_5\tau_a\pi^a\right)\psi\delta(R-r) + \frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}\left(\partial_\mu\pi\right)^2$$

In another model, proposed by G. E. Brown *et al.*, the meson kinetic term is pushed outside the bag, $\left[\frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \pi\right)^2\right] \Theta(r-R)$
In the Cloudy Bag Model the pion field is realized nonlinearly:

$$
\begin{aligned}
L_{CBM}(x) &= \left[\frac{i}{2}\bar{\psi} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi - B\right] \Theta(R-r) \\
&- \frac{1}{2}\bar{\psi} e^{i\tau_a \phi^a \gamma_5 / F_\pi} \psi \delta(R-r) + \frac{1}{2}\left(D_\mu \phi_a\right)^2 \Theta(r-R),
\end{aligned}
$$

where the covariant derivative is
$D_\mu \phi_a = \hat{\phi}_a \partial_\mu \phi + F_\pi \sin(\phi/F_\pi) \partial_\mu \hat{\phi}_a$, $\phi = \sqrt{\phi_a \phi^a}$, and $\hat{\phi}_a = \phi_a / \phi$
In all these extensions the basic dynamical element is the bag

Gell-Mann–Lévy σ-model

A simple way of introducing the chiral dynamics is the Gell-Mann–Lévy σ-model:

$$\begin{aligned} L_\sigma(x) &= \bar{\psi}[i\partial^\mu \gamma_\mu + m_c + g\,(\sigma + i\gamma_5 \tau^a \pi_a)]\psi \\ &+ \frac{1}{2}\,(\partial_\mu \sigma)^2 + \frac{1}{2}\,(\partial_\mu \pi_a)^2 - U(\sigma^2 + \pi_a^2) + m_\pi^2 F_\pi \sigma \end{aligned}$$

where g is the quark-meson coupling constant and

$$U = \frac{\lambda^2}{4}\left(\sigma^2 + \pi^2 - \nu^2\right)^2,$$

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2F_\pi^2}, \quad \nu^2 = \frac{3m_\sigma^2 - m_\pi^2}{m_\sigma^2 - m_\pi^2} F_\pi^2$$

Constituent quark mass: $M = m_c - g\langle\sigma\rangle = m_c + gF_\pi$

Birse-Banerjee model

Valence quarks + mean meson field, hedgehog solution:

$$| \, h \rangle = \frac{1}{\sqrt{2}} \left(| \, u \downarrow \rangle - | \, d \uparrow \rangle \right)$$

$$q_h = \begin{pmatrix} G(r) \\ i\sigma \cdot \hat{r} F(r) \end{pmatrix} | \, h \rangle, \quad \sigma = \sigma(r), \quad \pi^a = \hat{r}^a \phi(r)$$

Such a correlation of spin and isospin is required to achieve stationarity
E-L equations for G, F, σ, ϕ, solved numerically

Skyrmion

Quarks "integrated out":

$$L = \frac{F_\pi^2}{4} Tr(\partial_\mu U^\dagger, \partial^\mu U) + \frac{1}{32g^2} Tr[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

(T.H.R. Skyrme, 1962)
The topological current

$$J_B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} Tr(U^\dagger(\partial_\nu U) U^\dagger(\partial_\alpha U) U^\dagger(\partial_\beta U))$$

is identified with the baryon current
The stationary solution has the hedgehog form $U = e^{i\tau\cdot\hat{r}F(r)}$, with $F(0) = 0$ and $F(\infty) = \pi$. The baryon number is

$$B = \frac{1}{\pi}\left(f(\infty) - f(0) + \frac{1}{2}(\sin 2f(\infty) - \sin 2f(0))\right) = 1$$

Nambu–Jona-Lasinio model

Dynamical symmetry breaking
bosonization

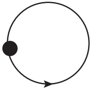
$$(i\partial^\mu\gamma_\mu - m)\psi = -G[(\bar{\psi}\psi)\psi + (\bar{\psi}i\gamma_5\tau^a\psi)i\gamma_5\tau^a\psi] = 0$$

Equivalent Lagrangian

$$L_{NJL}(x) = \bar{\psi}(i\partial^\mu\gamma_\mu - m)\psi + [(\bar{\psi}\psi)S + (\bar{\psi}i\gamma_5\tau^a\psi)P_a] - \frac{1}{2G}\left(S^2 + P^2\right)$$

yields

$$\begin{aligned}(i\partial^\mu\gamma_\mu - m + S + i\gamma_5\tau^a P_a)\psi &= 0\\ S &= G\bar{\psi}\psi\\ P_a &= G\bar{\psi}i\gamma_5\tau^a\psi\end{aligned}$$

The gap equation (for $m_c = 0$): $M = G\langle\bar{\psi}\psi\rangle =$ 

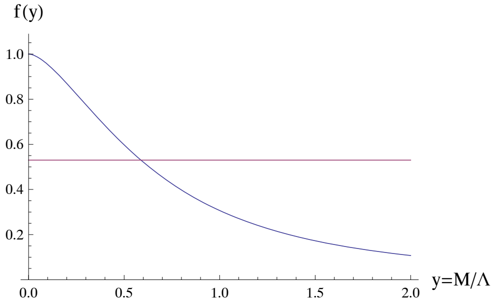
$$M = GN_cN_f \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \mathrm{Tr}(iS(p)) = GN_cN_f \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{4M}{k^2+M^2},$$

where $N_c = 3$, $N_f = 2$, and Λ denotes the cut-off in k^2. Then

$$M = \frac{GN_cN_f}{4\pi^2} M \left(\Lambda^2 - M^2 \ln\left(\Lambda^2/M^2+1\right)\right).$$

Trivial solution: $M = 0$. A nontrivial solution emerges for sufficiently large values G and Λ. The equation is

$$\begin{aligned} 1 &= \frac{GN_cN_f\Lambda^2}{4\pi^2} f(y), \\ f(y) &= 1 + y^2 \ln(y/(1+y)), \quad y = M/\Lambda \end{aligned}$$



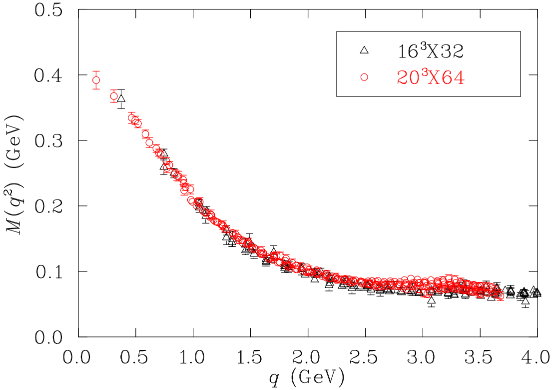
When the condition

$$\frac{4\pi^2}{GN_cN_f\Lambda^2} < 1$$

is satisfied, the gap equation has a solution and a nonzero quark mass M is generated (dynamical symmetry breaking). This nontrivial solution corresponds to the ground state of the theory (vacuum).

Quarks from lattices

$S(p) = \frac{1}{\not p - m} \to \frac{Z(p^2)}{\not p - M(p^2)}$ – nonperturbative effects



[Bowman et al., 2005] (Euclidean q, $q^2 = -p^2$)

Summary

- Hard dynamics – short range, asymptotic freedom, partons, perturbative methods
- Soft dynamics – long range, confinement, bags, strings, Regge trajectories, large-N_c expansion, simple patters in high-lying spectra
- Soft dynamics – intermediate range, spontaneously broken chiral symmetry as a key element, chiral perturbation theory, dynamical chiral symmetry breaking, chiral quark models, predictions for the pion and its interactions with gauge bosons, baryons as solitons, prediction of the Θ^+
- Nonperturbative aspects very are difficult, no exact analytic methods available
- Lattice QCD measurements, computer simulations provide valuable information for numerous observables, also those very difficult to measure, designed to work in the nonperturbative regime,
- New ideas: AdS/CFT correspondence, holographic QCD . . .

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